



Cambridge International AS & A Level

CANDIDATE NAME



CENTRE NUMBER

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MATHEMATICS

9709/33

Paper 3 Pure Mathematics 3

May/June 2024

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Any blank pages are indicated.





1 Solve the equation $8^{3-6x} = 4 \times 5^{-2x}$. Give your answer correct to 3 decimal places.

[4]

A series of horizontal dotted lines provided for the student to write their answer.

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2 Find the exact coordinates of the stationary point of the curve $y = e^{2x} \sin 2x$ for $0 \leq x \leq \frac{1}{2}\pi$. [5]

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3 The square roots of $24 - 7i$ can be expressed in the Cartesian form $x + iy$, where x and y are real and exact.

By first forming a quartic equation in x or y , find the square roots of $24 - 7i$ in exact Cartesian form. [5]

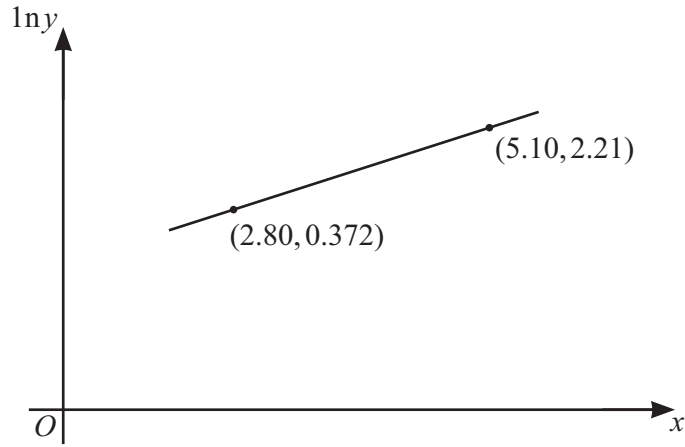
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The variables x and y satisfy the equation $ky = e^{cx}$, where k and c are constants. The graph of $\ln y$ against x is a straight line passing through the points $(2.80, 0.372)$ and $(5.10, 2.21)$, as shown in the diagram.

Find the values of k and c . Give each value correct to 2 significant figures. [4]

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5 Express $\frac{6x^2 - 2x + 2}{(x - 1)(2x + 1)}$ in partial fractions. [5]

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6 (a) On an Argand diagram shade the region whose points represent complex numbers z which satisfy both the inequalities $|z - 4 - 3i| \leq 2$ and $\arg(z - 2 - i) \geq \frac{1}{3}\pi$. [5]

(b) Calculate the greatest value of $\arg z$ for points in this region. [2]

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7 Let $f(x) = 8x^3 + 54x^2 - 17x - 21$.

(a) Show that $x + 7$ is a factor of $f(x)$. [1]

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(b) Find the quotient when $f(x)$ is divided by $x + 7$. [2]

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(c) Hence solve the equation

$$8 \cos^3 \theta + 54 \cos^2 \theta - 17 \cos \theta - 21 = 0,$$

for $0^\circ \leq \theta \leq 360^\circ$.

[3]

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(b) Hence find the exact value of $\int_0^{\frac{1}{12}\pi} \frac{3}{(3 \cos 2x - \sqrt{3} \sin 2x)^2} dx$, simplifying your answer. [5]

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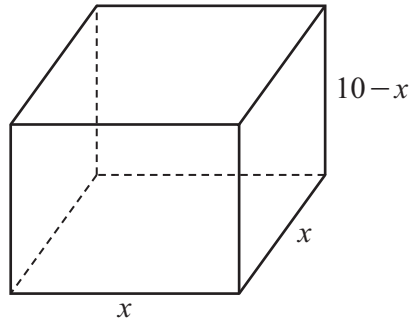
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A container in the shape of a cuboid has a square base of side x and a height of $(10 - x)$. It is given that x varies with time, t , where $t > 0$. The container decreases in volume at a rate which is inversely proportional to t .

When $t = \frac{1}{10}$, $x = \frac{1}{2}$ and the rate of decrease of x is $\frac{20}{37}$.

(a) Show that x and t satisfy the differential equation

$$\frac{dx}{dt} = \frac{-1}{2t(20x - 3x^2)}. \quad [5]$$

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10 The equations of two straight lines are

$$\mathbf{r} = \mathbf{i} + \mathbf{j} + 2a\mathbf{k} + \lambda(3\mathbf{i} + 4\mathbf{j} + a\mathbf{k}) \quad \text{and} \quad \mathbf{r} = -3\mathbf{i} - \mathbf{j} + 4\mathbf{k} + \mu(-\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}),$$

where a is a constant.

- (a) Given that the acute angle between the directions of these lines is $\frac{1}{4}\pi$, find the possible values of a . [6]

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(b) Given instead that the lines intersect, find the value of a and the position vector of the point of intersection. [5]

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11 Use the substitution $2x = \tan \theta$ to find the exact value of

$$\int_0^{\frac{1}{2}} \frac{12}{(1+4x^2)^2} dx .$$

Give your answer in the form $a + b\pi$, where a and b are rational numbers. [9]

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Additional page

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