Cambridge International AS & A Level

MATHEMATICS **9709/11**

Paper 1 Pure Mathematics 1 **May/June 2024**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2024 series for most Cambridge IGCSE, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1: Marks must be awarded in line with: • the specific content of the mark scheme or the generic level descriptors for the question • the specific skills defined in the mark scheme or in the generic level descriptors for the question • the standard of response required by a candidate as exemplified by the standardisation scripts.
GENERIC MARKING PRINCIPLE 2: Marks awarded are always **whole marks** (not half marks, or other fractions).
GENERIC MARKING PRINCIPLE 3: Marks must be awarded **positively**: • marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate • marks are awarded when candidates clearly demonstrate what they know and can do • marks are not deducted for errors • marks are not deducted for omissions • answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.
GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles

1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.

2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.

3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.

4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).

5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.

6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1(a)	$3(y-2)^2 - 27$ or $a = -2,\ b = -27$	**B1 B1**	
		2	
1(b)	$(x^2-2)^2 = 9$ leading to $x^2 - 2 = \pm 3$	**M1**	Must be x^2 unless substitution is clear.
	$x^2 = -1$ or $x^2 = 5$	**M1**	Allow omission of -1 if ±3 seen.
	$x = \pm\sqrt{5}$	**A1**	**B1 SC** if M1M1 not awarded. Ignore ± i, i, –i, √–1. Use of calculator with no working scores 0/3.
	Alternative method for Question 1(b)		
	$3x^4 - 12x^2 - 15 = 0$ leading to $3(x^2-5)(x^2+1)[=0]$	**(M1)**	
	$x^2 = -1$ or $x^2 = 5$	**(M1)**	Allow omission of –1 if factors seen. Factorising or other valid method.
	$x = \pm\sqrt{5}$	**(A1)**	**B1 SC** if M1M1 not scored. Ignore ± i, i, –i, √–1. Use of calculator with no working scores 0/3.
		3	

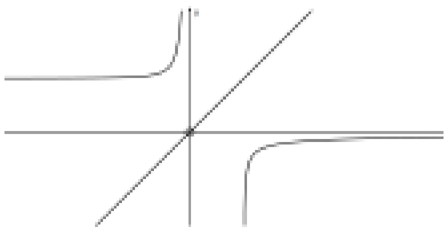
Question	Answer	Marks	Guidance
2(a)	{Stretch} {factor 3} { in y-direction}	**B2,1,0**	2 out of 3 scores B1.
	{Translation} $\begin{pmatrix} \{0\} \\ \{-2\} \end{pmatrix}$	**B2,1,0**	Accept shift.
	Alternative Method for Question 2(a)		
	{Translation} $\begin{pmatrix} \{0\} \\ \left\{-\dfrac{2}{3}\right\} \end{pmatrix}$	**(B2,1,0)**	2 out of 3 scores B1. Accept shift.
	{Stretch} {factor 3} { in y-direction}	**(B2,1,0)**	
		4	
2(b)	$[\mathrm{f}(x)] = \{-3\sin x\}\{-2\}$	**B1 B1**	No marks awarded if extra terms seen.
		2	

Question	Answer	Marks	Guidance
3(a)	$20 \times 27 \times a^3 [=160]$	**M1**	Allow 6C3 $\times$ $3^3 \times a^3 [=160]$. Accept $540a^3$ with no other working for M1.
	$[a] = \dfrac{2}{3}$	**A1**	Allow 0.667 AWRT. **SC B1** is $a = \dfrac{2}{3}$ with no other working.
		2	

Question	Answer	Marks	Guidance
3(b)	Coefficient of x^2 is $15\times81\times\left(their\frac{2}{3}\right)^2\ [=540]$	B1 FT	May be in a list. 6C2 and 3^4 must be evaluated but may be implied by later work. Condone 540 with no working.
	$160\times1-2\times their\ 540$	M1	
	$=-920$	A1	Condone $-920x^3$.
		3	

Question	Answer	Marks	Guidance
4(a)	[k] = 4.00063	B1	CAO
		1	
4(b)	[Gradient AE] = 6.3566	B1	CAO
		1	
4(c)	Suggests that $[f'(2)]=6.25$	B1	CAO
		1	

Question	Answer	Marks	Guidance
5(a)	$\frac{\sin^2 x - \cos x - 1}{1 + \cos x} = \frac{1 - \cos^2 x - \cos x - 1}{1 + \cos x}$ or $\frac{-\cos^2 x - \cos x}{1 + \cos x}$	**M1**	For use of $\sin^2 x + \cos^2 x = 1$. Allow use of s, c, t or omission of x throughout.
	$= \frac{-\cos x(1 + \cos x)}{1 + \cos x}$	**M1**	For factorising.
	$= -\cos x$	**A1**	
		3	
5(b)	$-\frac{1}{2}\cos x = \frac{1}{4} \Rightarrow x = \cos^{-1}\left(-\frac{1}{2}\right)$	**M1**	
	$x = 120°$ or $x = 240°$	**A1**	
		A1 FT	FT for 360 – *their* answer. A1 A0 if extra solution(s) in range. **SC B1** if answer in radians for both $\frac{2\pi}{3}$, $\frac{4\pi}{3}$.
		3	

Question	Answer	Marks	Guidance
6(a)		**B1**	For curve in correct quadrant.
		B1	Fully correct including line $y = x$. Horizontal asymptote closer to x axis than vertical asymptote is to y axis.
		2	

Question	Answer	Marks	Guidance
6(b)	$x = \frac{2}{y^2} + 4$ leading to $y^2(x-4) = 2$ or $y^2 = \frac{2}{(x-4)}$	M1	Allow *x* and *y* swapped around.
	$y^2 = \frac{2}{(x-4)}$ leading to $y = [\pm]\sqrt{\frac{2}{x-4}}$ or $x = [\pm]\sqrt{\frac{2}{y-4}}$	M1	
	$[\mathrm{f}^{-1}(x)] = -\sqrt{\frac{2}{x-4}}$	A1	
		3	
6(c)	$[x] = -2$	B1	
		1	
6(d)	Because f^{-1} is always negative and f is always positive or curves do not intersect	B1	Accept other correct answers e.g. 'f is only defined for positive values of *x* and f^{-1} is only defined for negative values of *x*' or 'domains do not overlap' or 'the *y* values cannot be the same' or 'the *x* values cannot be the same'.
		1	

Question	Answer	Marks	Guidance
7(a)	Angle $\theta = \frac{\pi}{2} - \cos^{-1}\frac{10}{15}$ or $\sin^{-1}\frac{10}{15} = 0.7297$	B1	Condone working in degrees if converted to radians at the end. AG
		1	

Question	Answer	Marks	Guidance
7(b)	$BC = \sqrt{15^2 - 10^2}$ $[= 11.18\ldots \text{ or } 5\sqrt{5}]$	B1	
	Arc AB = $15 \times 0.7297 [= 10.9455]$	B1	
	Perimeter = *their BC* + *their* arc AB + 25 + 5π	M1	
	Perimeter = 62.8	A1	AWRT
	Area sector AOB = $\frac{1}{2} \times 15^2 \times 0.7297 [= 82.09]$	B1	
	Area = $\frac{1}{2} \times 10 \times$ *their BC* + *their* sector AOB + $\frac{\pi}{4} \times 10^2$	M1	
	Area = 217	A1	AWRT
		7	

Question	Answer	Marks	Guidance
8(a)	$2(4p-1)=25+13-p$	***M1**	
	$p=\frac{40}{9}$	**A1**	
	$d=\left(4\times\frac{40}{9}-1\right)-25\left[=-\frac{74}{9}\right]$	**DM1**	Using *their p* to find *d*.
	10^{th} term = $25+9d=-49$	**A1**	
	Alternative Method for first 3 marks of Question 8(a)		
	$d=4p-26$, $d=14-5p$, p + 2d = -12 Any two	**(*M1)**	Allow unsimplified or equivalent.
	Solving simultaneously to find *p* or d	**(DM1)**	
	$\left[p=\frac{40}{9}\right]$, $d=-\frac{74}{9}$	**(A1)**	
		4	

Question	Answer	Marks	Guidance
8(b)	$(4q-1)^2 = 25(13-q) \Rightarrow 16q^2 + 17q - 324[=0]$	M1	
	$(q-4)(16q+81)[=0]$ leading to $[\Rightarrow q=4]$	M1	Solve 3 term quadratic with real solutions.
	$[r=]\frac{3}{5}$	A1	Ignore $\frac{-17}{20}$.
	Sum to infinity $= \frac{25}{1-\frac{3}{5}} = \frac{125}{2}$	A1	Ignore extra solution. **SC B1** if no method shown for solving quadratic.
	Alternative Method for Question 8(b)		
	$25r = 4q - 1$, $25r^2 = 13 - q$ leading to $100r^2 + 25r - 51 = 0$	(M1)	
	$(5r-3)(20r+17)=0$	(M1)	Solve 3 term quadratic with real solutions.
	$r = \frac{3}{5}$	(A1)	Ignore $\frac{-17}{20}$.
	Sum to infinity $= \frac{25}{1-\frac{3}{5}} = \frac{125}{2}$	(A1)	Ignore extra solution. **SC B1** if no method shown for solving quadratic.
		4	

Question	Answer	Marks	Guidance
9	Volume of cylinder = $\pi \times 1^2 \times \frac{7}{5} = \frac{7}{5}\pi$	**B1**	May be done using $\int_1^{2.4} 1$. This would be the only mark available if candidate integrates *y*.
	Volume under curve $= [\pi]\int \frac{1}{(5x-4)^{\frac{2}{3}}}\,dx$	**M1**	No further marks available if $\int y$.
	$= [\pi]\left\{\frac{3}{5}\right\}\left\{(5x-4)^{\frac{1}{3}}\right\}$	**B1 B1**	Calculator used for integration scores no further marks.
	$= [\pi]\frac{3}{5}\left(8^{\frac{1}{3}} - 1\right) \quad \left[= \frac{3}{5}\pi\right]$	**M1**	Uses limits 1, 2.4 in an integral of y^2.
	Volume = $\frac{7}{5}\pi - \frac{3}{5}\pi = \frac{4}{5}\pi$	**A1**	**SC B1** if the only error is not showing substitution.
		6	

Question	Answer	Marks	Guidance
10	$(x-3)^2+y^2=18$ $y=mx-9$ leading to $(x-3)^2+(mx-9)^2=18$	M1	Finding equation of tangent and substituting into circle equation. Must be $mx-9$.
	$x^2-6x+9+m^2x^2-18mx+81=18$ leading to $(m^2+1)x^2-(6+18m)x+72[=0]$	M1	Brackets expanded and all terms collected on one side of the equation. May be implied in the discriminant. *m* cannot be numeric.
	$(6+18m)^2-4(m^2+1)\times 72[=0]$	*M1	Use of b^2-4ac. Not in quadratic formula. *m* cannot be numeric, *c* must be numeric.
	$36m^2+216m-252[=0]$ $[\text{leading to } m^2+6m-7=0]$	DM1	Simplifies to 3 term quadratic.
	$m=1$ or $m=-7$	A1	Condone no method for solving quadratic shown.
	$m=1$ leading to $2x^2-24x+72=0$ leading to $x=6$	DM1	Must be correct *x* for *their* quadratic.
	$m=-7$ leading to $50x^2+120x+72=0$ leading to $x=-\frac{6}{5}$	DM1	Must be correct *x* for *their* quadratic.
	$(6,-3),\left(-\frac{6}{5},-\frac{3}{5}\right)$	A1	

Question	Answer	Marks	Guidance
10	**Alternative Method 1 for first 4 marks of Question 10**		
	$\frac{\lvert 3m-1(0)-9\rvert}{\sqrt{m^2+1}}$	**(M1)**	Use of the formula for the length of a perpendicular from a point to a line.
	$\frac{\lvert 3m-1(0)-9\rvert}{\sqrt{m^2+1}} = \sqrt{18}$	**(M1)**	Equates length of a perpendicular from a point to a line to the radius.
	$(3m-9)^2 = 18(m^2+1)$	**(M1)**	Squares and clears the fraction.
	$9m^2 - 54m + 81 = 0$ $\left[\text{leading to } m^2+6m-7=0\right]$	**(M1)**	
	Alternative Method 2 for first 3 marks of Question 10		
	$(3-x)(9+6x-x^2)^{-1/2} = m$	**(M1)**	OE Differentiates implicitly or otherwise and equates $\frac{dy}{dx}$ to m.
	$(1+m^2)x^2 - 6(1+m^2)x + 9(1-m^2)[=0]$	**(M1)**	Brackets expanded and all terms collected on one side of the equation. May be implied in the discriminant.
	$36(1+m^2)^2 - 4(1+m^2) \times 9(1-m^2)[=0]$	**(M1)**	Use of b^2-4ac.
		8	

Question	Answer	Marks	Guidance
11(a)	$\frac{dy}{dx} = -\frac{12}{x^4} + \frac{3}{x^2}$	B1	
	$\frac{dy}{dx} = -\frac{12}{x^4} + \frac{3}{x^2} = 0$ leading to $3x^4 - 12x^2 = 0$ or $-12 + 3x^2 = 0$	M1	Set = 0 or uses $<$, $\leqslant$ and simplifies. Must be from $\frac{dy}{dx} = \frac{A}{x^4} + \frac{B}{x^2}$.
	$3x^2\left(x^2 - 4\right) = 0$ leading to $x = \pm 2$ only	A1	**SC B1** for $x = \pm 2$ if M0 scored.
	$-2 < x < 0$ and $0 < x < 2$ or (-2, 0) and (0, 2) or $-2 < x < 2$ and $x \neq 0$	B1FT	Allow and/or.
		B1FT	Allow $-2 \leqslant x < 0$ and / or $0 < x \leqslant 2$ but only B1B0 if 0 included in either or both. Allow [–2, 0) and (0, 2]. Allow B1B0 for $-2 < x < 2$ or (–2, 2). Must be from $\frac{dy}{dx} = \frac{A}{x^4} + \frac{B}{x^2}$.
		5	B marks only available if $\frac{dy}{dx} = \frac{A}{x^4} + \frac{B}{x^2}$.

Question	Answer	Marks	Guidance
11(b)	[At $x = 1$] $y = 3$ and $m\tan = -9$	*M1	Using *their* $\frac{dy}{dx}$.
	$m\text{norm} = -\frac{1}{-9} = \frac{1}{9}$	DM1	
	Equation of normal is $y - 3 = \frac{1}{9}(x-1)$ $\left[\text{leading to } y = \frac{1}{9}x + \frac{26}{9}\right]$	A1	
	At $x = -1, y = 1, m = -9$	M1	
	Equation of tangent is $y - 1 = -9(x+1)$ $[\text{leading to } y = -9x - 8]$	A1	
	Meet when $\frac{1}{9}x + \frac{26}{9} = -9x - 8$ $\left[\text{leading to } x = -1.19512\ldots, \frac{-49}{41}\right]$	M1	Equates *their* tangent and *their* normal.
	Area = $\frac{1}{2} \times \textit{their}\, 1.19512\ldots \times \textit{their}\left(\frac{26}{9} + 8\right)$	M1	If $\int y_2 - y_1$ is used integration must be correct and substitution shown.
	6.51	A1	AWRT Accept fraction wrt 6.51
		8	