



Cambridge International AS & A Level

CANDIDATE
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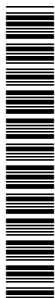
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MATHEMATICS

9709/32

Paper 3 Pure Mathematics 3

October/November 2023

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Any blank pages are indicated.

BLANK PAGE

1 (a) Sketch the graph of $y = |4x - 2|$.

[1]

(b) Solve the inequality $1 + 3x < |4x - 2|$.

[4]

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2 The parametric equations of a curve are

$$x = (\ln t)^2, \quad y = e^{2-t^2},$$

for $t > 0$.

Find the gradient of the curve at the point where $t = e$, simplifying your answer.

[4]

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- 3 The polynomial $2x^3 + ax^2 - 11x + b$ is denoted by $p(x)$. It is given that $p(x)$ is divisible by $(2x - 1)$ and that when $p(x)$ is divided by $(x + 1)$ the remainder is 12.

Find the values of a and b .

[5]

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- 4 (a) On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying the inequalities $|z - 4 - 3i| \leq 2$ and $\operatorname{Re} z \leq 3$. [4]

- (b) Find the greatest value of $\arg z$ for points in this region. [2]

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5 Find the exact value of $\int_0^6 \frac{x(x+1)}{x^2+4} dx.$ [6]

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- 6 (a) By sketching a suitable pair of graphs, show that the equation

$$\cot x = 2 - \cos x$$

has one root in the interval $0 < x \leq \frac{1}{2}\pi$.

[2]

- (b) Show by calculation that this root lies between 0.6 and 0.8.

[2]

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- (c) Use the iterative formula $x_{n+1} = \tan^{-1}\left(\frac{1}{2 - \cos x_n}\right)$ to determine the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

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(b) Hence solve the equation

$$\cos 3\theta + \cos \theta \cos 2\theta = \cos^2 \theta$$

for $0^\circ \leq \theta \leq 180^\circ$.

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8 It is given that $\frac{2+3ai}{a+2i} = \lambda(2-i)$, where a and λ are real constants.

(a) Show that $3a^2 + 4a - 4 = 0$. [4]

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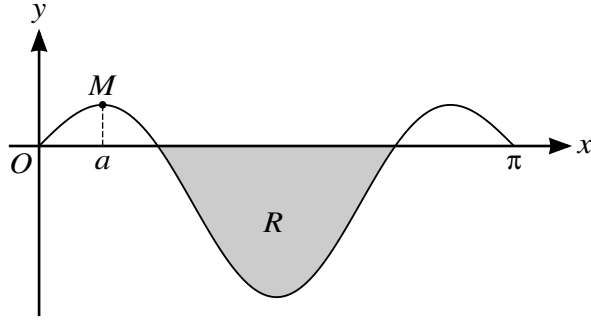
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The diagram shows the curve $y = \sin x \cos 2x$, for $0 \leq x \leq \pi$, and a maximum point M , where $x = a$. The shaded region between the curve and the x -axis is denoted by R .

(a) Find the value of a correct to 2 decimal places. [5]

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10 The equations of the lines l and m are given by

$$l: \mathbf{r} = \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} \quad \text{and} \quad m: \mathbf{r} = \begin{pmatrix} 6 \\ -3 \\ 6 \end{pmatrix} + \mu \begin{pmatrix} -2 \\ 4 \\ c \end{pmatrix},$$

where c is a positive constant. It is given that the angle between l and m is 60° .

(a) Find the value of c .

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- (b) Show that the length of the perpendicular from $(6, -3, 6)$ to l is $\sqrt{11}$. [5]

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11 The variables x and y satisfy the differential equation

$$x^2 \frac{dy}{dx} + y^2 + y = 0.$$

It is given that $x = 1$ when $y = 1$.

- (a) Solve the differential equation to obtain an expression for y in terms of x . [8]

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(b) State what happens to the value of y when x tends to infinity. Give your answer in an exact form. [1]

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Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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