



# Cambridge International AS & A Level

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**MATHEMATICS**

**9709/31**

Paper 3 Pure Mathematics 3

**October/November 2023**

**1 hour 50 minutes**

You must answer on the question paper.

You will need: List of formulae (MF19)

## INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

## INFORMATION

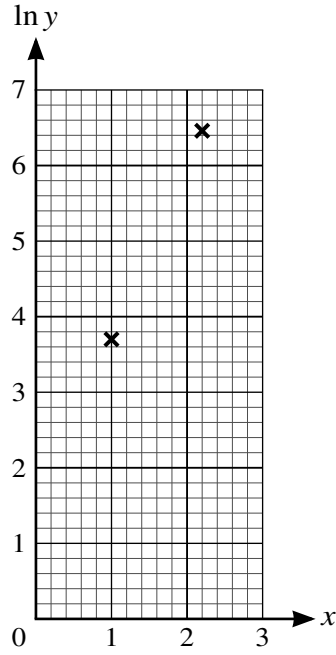
- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.



- 2 On an Argand diagram, shade the region whose points represent complex numbers  $z$  satisfying the inequalities  $|z - 2i| \leq |z + 2 - i|$  and  $0 \leq \arg(z + 1) \leq \frac{1}{4}\pi$ . [4]

3



The variables  $x$  and  $y$  are related by the equation  $y = ab^x$ , where  $a$  and  $b$  are constants. The diagram shows the result of plotting  $\ln y$  against  $x$  for two pairs of values of  $x$  and  $y$ . The coordinates of these points are  $(1, 3.7)$  and  $(2.2, 6.46)$ .

Use this information to find the values of  $a$  and  $b$ . [4]

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4 The complex number  $u$  is defined by  $u = \frac{3 + 2i}{a - 5i}$ , where  $a$  is real.

(a) Express  $u$  in the Cartesian form  $x + iy$ , where  $x$  and  $y$  are in terms of  $a$ . [3]

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(b) Given that  $\arg u = \frac{1}{4}\pi$ , find the value of  $a$ . [2]

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5 (a) Given that

$$\sin\left(x + \frac{1}{6}\pi\right) - \sin\left(x - \frac{1}{6}\pi\right) = \cos\left(x + \frac{1}{3}\pi\right) - \cos\left(x - \frac{1}{3}\pi\right),$$

find the exact value of  $\tan x$ .

[4]

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6 The parametric equations of a curve are

$$x = \sqrt{t} + 3, \quad y = \ln t,$$

for  $t > 0$ .

(a) Obtain a simplified expression for  $\frac{dy}{dx}$  in terms of  $t$ . [3]

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(b) Hence find the exact coordinates of the point on the curve at which the gradient of the normal is  $-2$ . [3]

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- 8 (a) By sketching a suitable pair of graphs, show that the equation

$$\sqrt{x} = e^x - 3$$

has only one root.

[2]

- (b) Show by calculation that this root lies between 1 and 2.

[2]

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