Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER | | | | | |

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1395628472

MATHEMATICS **9709/63**

Paper 6 Probability & Statistics 2 **May/June 2023**

1 hour 15 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

1 A random variable X has probability density function f, where

$$f(x) = \begin{cases} \frac{3}{2}(1 - x^2) & 0 \leqslant x \leqslant 1, \\ 0 & \text{otherwise.} \end{cases}$$

Find $E(X)$. [3]

2 A club has 264 members, numbered from 1 to 264. Donash wants to choose a random sample of members for a survey. In order to choose the members for the sample he uses his calculator to generate random digits. His first 20 random digits are as follows.

10612 11801 21473 22759

(a) The numbers of the first two members in the sample are 106 and 121.

Write down the numbers of the next two members in the sample. [2]

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(b) To obtain the numbers for members after the 4th member, Donash starts with the second random digit, 0, and obtains the numbers 061 and 211.

Explain why this method will not produce a random sample. [1]

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3 In a random sample of 100 students at Luciana's college, x students said that they liked exams. Luciana used this result to find an approximate 90% confidence interval for the proportion, p, of all students at her college who liked exams. Her confidence interval had width 0.157 92.

(a) Find the two possible values of x. [4]

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Suzma independently took another random sample and found another approximate 90% confidence interval for p.

(b) Find the probability that neither of the two confidence intervals contains the true value of p. [1]

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4 The mass, in tonnes, of steel produced per day at a factory is normally distributed with mean 65.2 and standard deviation 3.6. It can be assumed that the mass of steel produced each day is independent of other days. The factory makes \$50 profit on each tonne of steel produced.

Find the probability that the total profit made in a randomly chosen 7-day week is less than \$22 000. [6]

5 Last year the mean time for pizza deliveries from Pete's Pizza Pit was 32.4 minutes. This year the time, t minutes, for pizza deliveries from Pete's Pizza Pit was recorded for a random sample of 50 deliveries. The results were as follows.

$$n = 50 \qquad \Sigma t = 1700 \qquad \Sigma t^2 = 59\,050$$

(a) Find unbiased estimates of the population mean and variance. [3]

(b) Test, at the 2% significance level, whether the mean delivery time has changed since last year. [5]

(c) Under what circumstances would it **not** be necessary to use the Central Limit Theorem in answering **(b)**? [1]

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6 It is known that 1 in 5000 people in Atalia have a certain condition. A random sample of 12 500 people from Atalia is chosen for a medical trial. The number having the condition is denoted by X.

(a) Use an appropriate approximating distribution to find $P(X \leqslant 3)$. [3]

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(b) Find the values of $E(X)$ and $Var(X)$, and explain how your answers suggest that the approximating distribution used in **(a)** is likely to be appropriate. [2]

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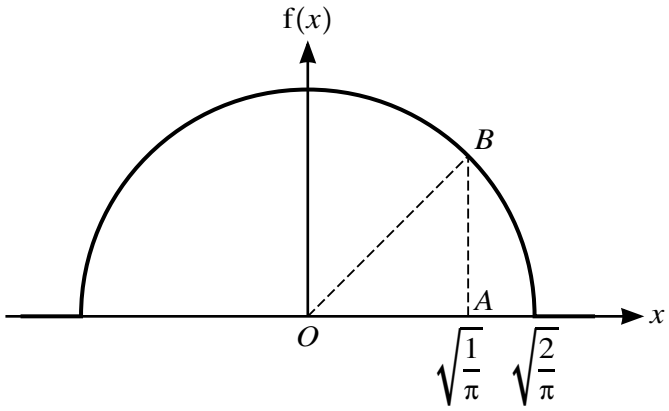

A random variable X has probability density function f, where the graph of $y = \mathrm{f}(x)$ is a semicircle with centre (0, 0) and radius $\sqrt{\frac{2}{\pi}}$, entirely above the x-axis. Elsewhere $\mathrm{f}(x) = 0$ (see diagram).

(a) Verify that f can be a probability density function. [2]

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A and B are the points where the line $x = \sqrt{\frac{1}{\pi}}$ meets the x-axis and the semicircle respectively.

(b) Show that angle AOB is $\frac{1}{4}\pi$ radians and hence find $\mathrm{P}\left(X > \sqrt{\frac{1}{\pi}}\right)$. [6]

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8 A new light was installed on a certain footpath. A town councillor decided to use a hypothesis test to investigate whether the number of people using the path in the evening had increased.

Before the light was installed, the mean number of people using the path during any 20-minute period during the evening was 1.01.

After the light was installed, the total number, n, of people using the path during 3 randomly chosen 20-minute periods during the evening was noted.

(a) Given that the value of n was 6, use a Poisson distribution to carry out the test at the 5% significance level. [6]

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(b) Later a similar test, at the 5% significance level, was carried out using another 3 randomly chosen 20-minute periods during the evening.

Find the probability of a Type I error. [2]

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(c) State what is meant by a Type I error in this context. [1]

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(d) State, in context, what further information would be needed in order to find the probability of a Type II error. Do not carry out any further calculation. [2]

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Additional Page

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