



Cambridge International AS & A Level

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MATHEMATICS

9709/33

Paper 3 Pure Mathematics 3

May/June 2023

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages.

1 Solve the equation $\ln(x + 5) = 5 + \ln x$. Give your answer correct to 3 decimal places. [4]

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2 Find the quotient and remainder when $2x^4 - 27$ is divided by $x^2 + x + 3$. [3]

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- 3 On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying the inequalities $|z - 3 - i| \leq 3$ and $|z| \geq |z - 4i|$. [4]

4 The parametric equations of a curve are

$$x = \frac{\cos \theta}{2 - \sin \theta}, \quad y = \theta + 2 \cos \theta.$$

Show that $\frac{dy}{dx} = (2 - \sin \theta)^2$. [5]

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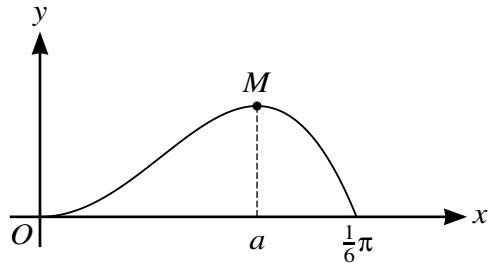
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The diagram shows the part of the curve $y = x^2 \cos 3x$ for $0 \leq x \leq \frac{1}{6}\pi$, and its maximum point M , where $x = a$.

(a) Show that a satisfies the equation $a = \frac{1}{3} \tan^{-1}\left(\frac{2}{3a}\right)$. [3]

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(b) Hence solve the equation

$$3 \cos 2\theta + 2 \cos(2\theta - 60^\circ) = 2.5$$

for $0^\circ < \theta < 180^\circ$.

[4]

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(b) Hence find the exact value of $\int_0^\pi \sin 2x e^{2 \cos x} dx$. [4]

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8 The variables x and y satisfy the differential equation

$$\frac{dy}{dx} = \frac{y^2 + 4}{x(y + 4)}$$

for $x > 0$. It is given that $x = 4$ when $y = 2\sqrt{3}$.

Solve the differential equation to obtain the value of x when $y = 2$. [8]

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- (b) The perpendicular from P meets line m at Q . The point R lies on PQ extended, with $PQ : QR = 2 : 3$.

Find the position vector of R .

[6]

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10 Let $f(x) = \frac{21 - 8x - 2x^2}{(1 + 2x)(3 - x)^2}$.

(a) Express $f(x)$ in partial fractions.

[5]

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11 The complex number z is defined by $z = \frac{5a - 2i}{3 + ai}$, where a is an integer. It is given that $\arg z = -\frac{1}{4}\pi$.

(a) Find the value of a and hence express z in the form $x + iy$, where x and y are real. [6]

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- (b) Express z^3 in the form $re^{i\theta}$, where $r > 0$ and $-\pi < \theta \leq \pi$. Give the simplified exact values of r and θ . [3]

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