Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

2139890131

MATHEMATICS **9709/22**

Paper 2 Pure Mathematics 2 **May/June 2023**

1 hour 15 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

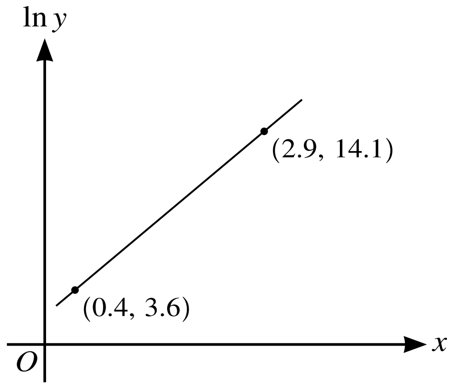
This document has **12** pages.

1 Solve the equation

$$\sec^2\theta + 5\tan^2\theta = 9 + 17\sec\theta$$

for $0° < \theta < 360°$. [5]

2



The variables x and y satisfy the equation $y = Ae^{(A-B)x}$, where A and B are constants. The graph of $\ln y$ against x is a straight line passing through the points (0.4, 3.6) and (2.9, 14.1), as shown in the diagram.

Find the values of A and B correct to 3 significant figures. [5]

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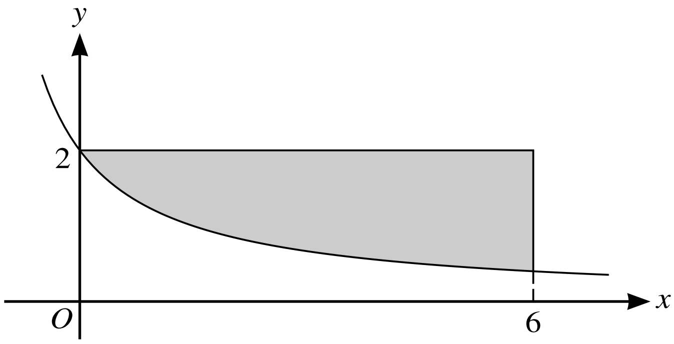
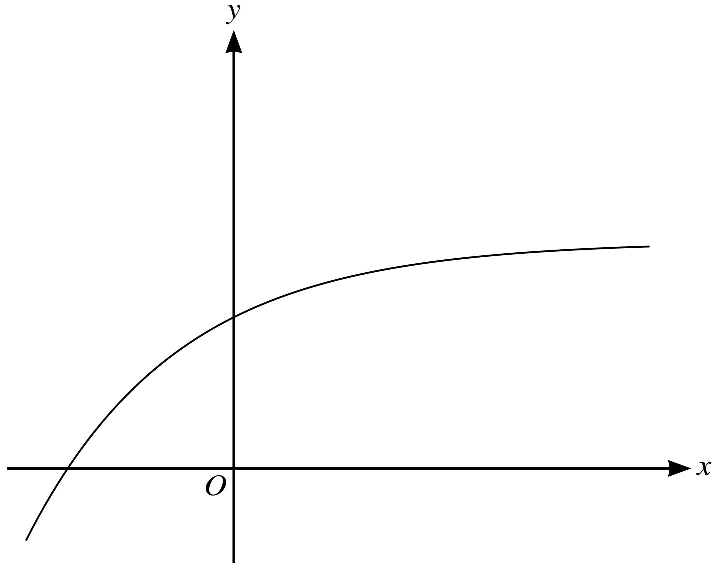


The diagram shows part of the curve $y = \dfrac{6}{2x+3}$. The shaded region is bounded by the curve and the lines $x = 6$ and $y = 2$.

Find the exact area of the shaded region, giving your answer in the form $a - \ln b$, where a and b are integers. [5]

4 **(a)**



The diagram shows the graph of $y = 3 - e^{-\frac{1}{2}x}$.

On the diagram, sketch the graph of $y = |5x - 4|$, and show that the equation $3 - e^{-\frac{1}{2}x} = |5x - 4|$ has exactly two real roots. [2]

It is given that the two roots of $3 - e^{-\frac{1}{2}x} = |5x - 4|$ are denoted by α and β, where $\alpha < \beta$.

(b) Show by calculation that α lies between 0.36 and 0.37. [2]

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(c) Use the iterative formula $x_{n+1} = \frac{1}{5}\left(7 - e^{-\frac{1}{2}x_n}\right)$ to find β correct to 4 significant figures. Give the result of each iteration to 6 significant figures. [3]

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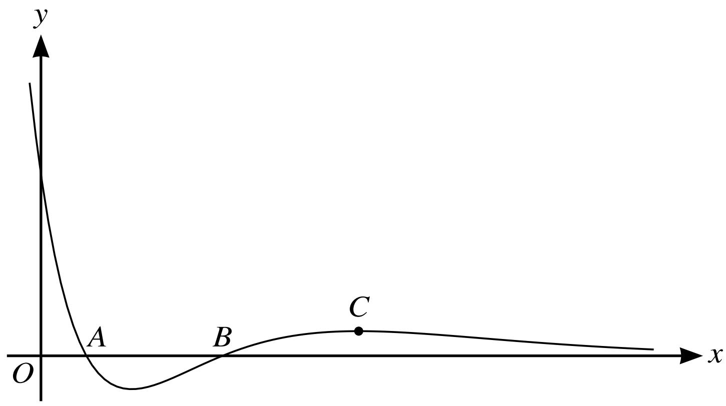

The diagram shows the curve with equation $y = e^{-\frac{1}{2}x}(x^2 - 5x + 4)$. The curve crosses the x-axis at the points A and B, and has a maximum at the point C.

(a) Find the exact gradient of the curve at B. [5]

(b) Find the exact coordinates of C. [4]

6 **(a)** Show that $4\sin\left(\theta + \frac{1}{3}\pi\right)\cos\left(\theta - \frac{1}{3}\pi\right) \equiv \sqrt{3} + 2\sin 2\theta$. [4]

(b) Find the exact value of $4\sin\frac{17}{24}\pi\cos\frac{1}{24}\pi$. [2]

(c) Find the exact value of $\int_0^{\frac{1}{8}\pi} 4\sin\left(2x + \frac{1}{3}\pi\right)\cos\left(2x - \frac{1}{3}\pi\right)\,dx$. [4]

7 A curve has parametric equations

$$x = \frac{2t+3}{t+2}, \qquad y = t^2 + at + 1,$$

where a is a constant. It is given that, at the point P on the curve, the gradient is 1.

(a) Show that the value of t at P satisfies the equation

$$2t^3 + (a+8)t^2 + (4a+8)t + 4a - 1 = 0.$$ [4]

(b) It is given that $(t + 1)$ is a factor of

$$2t^3 + (a + 8)t^2 + (4a + 8)t + 4a - 1.$$

Find the value of a. [2]

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(c) Hence show that P is the only point on the curve at which the gradient is 1. [3]

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Additional Page

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