Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

* 0 7 4 6 8 6 6 0 5 1 *

MATHEMATICS **9709/31**

Paper 3 Pure Mathematics 3 **October/November 2021**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS
- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION
- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages.

JC21 11_9709_31/RP

1 Solve the equation $4|5^x - 1| = 5^x$, giving your answers correct to 3 decimal places. [4]

2 **(a)** Express $5\sin x - 3\cos x$ in the form $R\sin(x - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$. Give the exact value of R and give α correct to 2 decimal places. [3]

(b) Hence state the greatest and least possible values of $(5\sin x - 3\cos x)^2$. [2]

3 The curve with equation $y = xe^{1-2x}$ has one stationary point.

(a) Find the coordinates of this point. [4]

(b) Determine whether the stationary point is a maximum or a minimum. [2]

4 Using the substitution $u = \sqrt{x}$, find the exact value of

$$\int_3^{\infty} \frac{1}{(x+1)\sqrt{x}}\, dx.$$ [6]

5 **(a)** Show that the equation

$$\cot 2\theta + \cot \theta = 2$$

can be expressed as a quadratic equation in $\tan \theta$. [3]

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(b) Hence solve the equation $\cot 2\theta + \cot \theta = 2$, for $0 < \theta < \pi$, giving your answers correct to 3 decimal places. [3]

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6 When $(a + bx)\sqrt{1 + 4x}$, where a and b are constants, is expanded in ascending powers of x, the coefficients of x and x^2 are 3 and -6 respectively.

Find the values of a and b. [6]

7 **(a)** Given that $y = \ln(\ln x)$, show that

$$\frac{dy}{dx} = \frac{1}{x \ln x}.$$ [1]

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The variables x and t satisfy the differential equation

$$x \ln x + t\frac{dx}{dt} = 0.$$

It is given that $x = e$ when $t = 2$.

(b) Solve the differential equation obtaining an expression for x in terms of t, simplifying your answer. [7]

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(c) Hence state what happens to the value of x as t tends to infinity. [1]

8 The constant a is such that $\int_1^a \frac{\ln x}{\sqrt{x}}\,dx = 6$.

(a) Show that $a = \exp\left(\frac{1}{\sqrt{a}} + 2\right)$. [5]

[$\exp(x)$ is an alternative notation for e^x.]

(b) Verify by calculation that a lies between 9 and 11. [2]

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(c) Use an iterative formula based on the equation in part **(a)** to determine a correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

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9 Two lines l and m have equations $\mathbf{r} = 3\mathbf{i} + 2\mathbf{j} + 5\mathbf{k} + s(4\mathbf{i} - \mathbf{j} + 3\mathbf{k})$ and $\mathbf{r} = \mathbf{i} - \mathbf{j} - 2\mathbf{k} + t(-\mathbf{i} + 2\mathbf{j} + 2\mathbf{k})$ respectively.

(a) Show that l and m are perpendicular. [2]

(b) Show that l and m intersect and state the position vector of the point of intersection. [5]

(c) Show that the length of the perpendicular from the origin to the line m is $\frac{1}{3}\sqrt{5}$. [4]

10 The complex number $1 + 2i$ is denoted by u. The polynomial $2x^3 + ax^2 + 4x + b$, where a and b are real constants, is denoted by $p(x)$. It is given that u is a root of the equation $p(x) = 0$.

(a) Find the values of a and b. [4]

(b) State a second complex root of this equation. [1]

(c) Find the real factors of $\text{p}(x)$. [2]

(d) **(i)** On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying the inequalities $|z - u| \leqslant \sqrt{5}$ and $\arg z \leqslant \frac{1}{4}\pi$. [4]

(ii) Find the least value of $\text{Im}\,z$ for points in the shaded region. Give your answer in an exact form. [1]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.