Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

8483881097

MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **February/March 2021**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.

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1 Solve the equation $\ln(x^3 - 3) = 3\ln x - \ln 3$. Give your answer correct to 3 significant figures. [3]

2 The polynomial $ax^3 + 5x^2 - 4x + b$, where a and b are constants, is denoted by $\mathrm{p}(x)$. It is given that $(x + 2)$ is a factor of $\mathrm{p}(x)$ and that when $\mathrm{p}(x)$ is divided by $(x + 1)$ the remainder is 2.

Find the values of a and b. [5]

3 By first expressing the equation $\tan(x + 45°) = 2\cot x + 1$ as a quadratic equation in $\tan x$, solve the equation for $0° < x < 180°$. [6]

4 The variables x and y satisfy the differential equation

$$(1 - \cos x)\frac{\mathrm{d}y}{\mathrm{d}x} = y \sin x.$$

It is given that $y = 4$ when $x = \pi$.

(a) Solve the differential equation, obtaining an expression for y in terms of x. [6]

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(b) Sketch the graph of y against x for $0 < x < 2\pi$. [1]

5 **(a)** Express $\sqrt{7}\sin x + 2\cos x$ in the form $R\sin(x + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. State the exact value of R and give α correct to 2 decimal places. [3]

(b) Hence solve the equation $\sqrt{7}\sin 2\theta + 2\cos 2\theta = 1$, for $0° < \theta < 180°$. [5]

6 Let $f(x) = \dfrac{5a}{(2x-a)(3a-x)}$, where a is a positive constant.

(a) Express $f(x)$ in partial fractions. [3]

(b) Hence show that $\int_a^{2a} \mathrm{f}(x)\,\mathrm{d}x = \ln 6$. [4]

7 Two lines have equations $\mathbf{r} = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix} + s \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix} + t \begin{pmatrix} 1 \\ -1 \\ 4 \end{pmatrix}$.

(a) Show that the lines are skew. [5]

(b) Find the acute angle between the directions of the two lines. [3]

8 The complex numbers u and v are defined by $u = -4 + 2\mathrm{i}$ and $v = 3 + \mathrm{i}$.

(a) Find $\frac{u}{v}$ in the form $x + \mathrm{i}y$, where x and y are real. [3]

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(b) Hence express $\frac{u}{v}$ in the form $re^{\mathrm{i}\theta}$, where r and θ are exact. [2]

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In an Argand diagram, with origin O, the points A, B and C represent the complex numbers u, v and $2u + v$ respectively.

(c) State fully the geometrical relationship between OA and BC. [2]

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(d) Prove that angle $AOB = \frac{3}{4}\pi$. [2]

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9 Let $f(x) = \dfrac{e^{2x} + 1}{e^{2x} - 1}$, for $x > 0$.

(a) The equation $x = f(x)$ has one root, denoted by a.

Verify by calculation that a lies between 1 and 1.5. [2]

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(b) Use an iterative formula based on the equation in part **(a)** to determine a correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

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(c) Find $f'(x)$. Hence find the exact value of x for which $f'(x) = -8$. [6]

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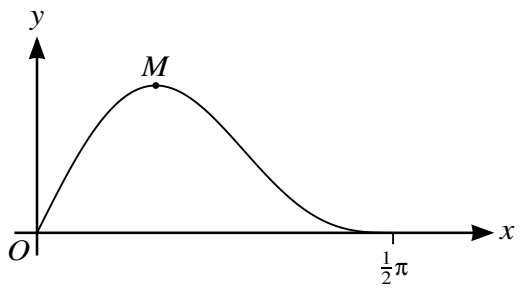

The diagram shows the curve $y = \sin 2x \cos^2 x$ for $0 \leqslant x \leqslant \frac{1}{2}\pi$, and its maximum point M.

(a) Using the substitution $u = \sin x$, find the exact area of the region bounded by the curve and the x-axis. [5]

(b) Find the exact x-coordinate of M. [6]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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