

Cambridge O Level

ADDITIONAL MATHEMATICS**4037/22**

Paper 2

October/November 2024

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2024 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

This document consists of **12** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics-Specific Marking Principles

- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method marks, awarded for a valid method applied to the problem.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.
- B** Mark for a correct result or statement independent of Method marks.

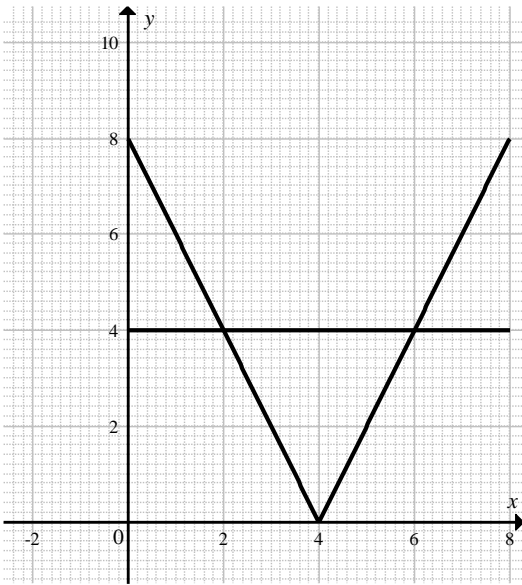
When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation 'dep' is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

Abbreviations

- awrt answers which round to
 cao correct answer only
 dep dependent
 FT follow through after error
 isw ignore subsequent working
 nfwf not from wrong working
 oe or equivalent
 rot rounded or truncated
 SC Special Case
 soi seen or implied

| Question | Answer | Marks | Partial marks |
|----------|---|-------------|---|
| 1 | Correct elimination of one unknown e.g. $\left(\frac{3}{2}\right)^4 \times \frac{1}{x} = \frac{27}{16}$ or $\left(\frac{3}{2}x\right)^4 \times \frac{1}{x^5} = \frac{27}{16}$ or $\frac{y^4}{\left(\frac{2y}{3}\right)^5} = \frac{27}{16}$ or $16\left(\frac{81}{16}x^4\right) = 27x^5$ or $16y^4 = 27\left(\frac{32}{243}y^5\right)$ oe | M1 | |
| | $x = 3$ and $y = \frac{9}{2}$ oe and no other solutions | A2 | A1 for $x = 3$ or $y = \frac{9}{2}$ oe |
| 2(a) | $\frac{d}{dx}\sqrt{1+2x} = (1+2x)^{-\frac{1}{2}}$ or $\frac{1}{2} \times (1+2x)^{-\frac{1}{2}} \times 2$ oe | B2 | B1 for $k(1+2x)^{-\frac{1}{2}}$ where k is a positive constant, $k \neq 1$ |
| | $x \times \text{their} \left(\frac{1}{2}(1+2x)^{-\frac{1}{2}} \times 2\right) + [1](1+2x)^{\frac{1}{2}}$ oe, isw | B1 | FT <i>their</i> $\frac{d}{dx}\sqrt{1+2x}$ |
| | Alternative $\frac{1}{2}(x^2 + 2x^3)^{-\frac{1}{2}} \times (2x + 6x^2)$ or $\frac{1}{2}(x^2(1+2x))^{-\frac{1}{2}} \times (2x^2 + 2x(1+2x))$ | (B3) | B2 for $\frac{1}{2}(x^2 + 2x^3)^{-\frac{1}{2}} \times (ax + bx^2)$ or $\frac{1}{2}(x^2(1+2x))^{-\frac{1}{2}} \times (ax + bx^2)$ where a and b are constants or B1 for $k(x^2 + 2x^3)^{-\frac{1}{2}}$ or $k(x^2(1+2x))^{-\frac{1}{2}}$ where k is a positive constant, soi |
| 2(b) | Uses <i>their</i> $4(1+2(4))^{-\frac{1}{2}} + (1+2(4))^{\frac{1}{2}}$ in an attempt at a small changes relationship | M1 | FT $x = 4$ substituted into <i>their</i> derivative |
| | $\frac{0.06}{\delta x} = \text{their} \left(4(1+2(4))^{-\frac{1}{2}} + (1+2(4))^{\frac{1}{2}}\right)$ oe | M1 | dep previous M1 FT <i>their</i> $\frac{13}{3}$ |
| | 0.0138 or 0.01384[6...] or $\frac{9}{650}$ oe nfw | A1 | |

| Question | Answer | Marks | Partial marks |
|----------|---|-----------|--|
| 2(c) | $\frac{x}{\sqrt{1+2x}} + \sqrt{1+2x} = 0$ oe and solves as far as $x = \dots$ | M1 | FT a derivative of the form $\frac{ax}{\sqrt{1+2x}} + b\sqrt{1+2x}$ or $\frac{a}{\sqrt{1+2x}} + b\sqrt{1+2x}$ or $\frac{ax+b}{\sqrt{1+2x}}$ oe |
| | $x = -\frac{1}{3}$ oe | A1 | |
| 3(a) | $8a - 12 - 6 + b = 0$ oe and $-a - 3 + 3 + b = 0$ oe and $a = 2, b = 2$ | B3 | B1 for $8a - 12 - 6 + b = 0$ oe B1 for $-a - 3 + 3 + b = 0$ oe |
| 3(b) | $[2x^3 - 3x^2 - 3x + 2 =]$ $(x - 2), (x + 1), (2x - 1)$ or $(x^2 - x - 2), (2x - 1)$ and $x = 2, \frac{1}{2}, -1$ OR $[2x^3 - 3x^2 - 3x + 2 =]$ $(x + 1), (2x^2 - 5x + 2)$ or $(x - 2), (2x^2 + x - 1)$ and correct factorisation or method of solution of the quadratic and $x = 2, \frac{1}{2}, -1$ OR $2 - 1 + x = -\left(\frac{-3}{2}\right)$ or $2 - 1 + x = \frac{3}{2}$ or for $2 \times -1 \times x = \frac{-2}{2}$ or $2 \times -1 \times x = -1$ and $x = 2, \frac{1}{2}, -1$ | B2 | B1 for $[2x^3 - 3x^2 - 3x + 2 =]$ $(x - 2)$ and $(x + 1)$ seen or $(x^2 - x - 2)$ seen or $(x + 1), (2x^2 - 5x + 2)$ or $(x - 2), (2x^2 + x - 1)$ B1 for $2 - 1 + x = -\left(\frac{-3}{2}\right)$ or $\frac{3}{2}$ or for $2 \times -1 \times x = \frac{-2}{2}$ or -1 |

| Question | Answer | Marks | Partial marks |
|----------|---|-----------|---|
| 4 | <p>Correct intersecting graphs</p>  | 3 | <p>$y = 2x - 8$: M1 for an attempt to draw the sections from (0, 8) to (2, 4) and (6, 4) to (8, 8) with at least one side accurate or for \vee shape with vertex at (4, 0) A1 for correct graph B1 for $y = 4$ drawn</p> |
| | critical values: 2, 6 | M1 | dep on 3 marks awarded for intersecting graphs |
| | $x < 2, x > 6$ mark final answer | A1 | |
| 5(a) | <p>Correctly derives correct equation free of logarithms e.g. $x^9 = 16^{18}$ or $x = 16^2$ oe or $x^9 = 2^{72}$ or $x = 2^8$ oe or $x^{\frac{9}{4}} = 2^{18}$ oe</p> | M3 | <p>M2 for correctly changing to consistent bases and correct use of one other log law or correct use of $\log_a a = 1$ in a correct equation e.g. $\frac{\log_{16} x^2}{\frac{1}{4}} + \log_{16} x = 18$ oe or $\log_2 x^2 + \frac{\log_2 x}{4} = 18$ oe or $\frac{\log_x x^2}{\log_x 2} + \frac{\log_x x}{4 \log_x 2} = 18$ or M1 for correctly changing to consistent bases or correct use of one other log law or correct use of $\log_a a = 1$ in a correct equation</p> |
| | $x = 256$ nfw | A1 | |

| Question | Answer | Marks | Partial marks |
|---|---|-----------------------|--|
| 5(b) | $e^{4x+2} - 3e^{2x+1} - 10 [= 0]$ or $(e^{2x+1})^2 - 3e^{2x+1} - 10 [= 0]$ | B1 | |
| | Solves or factorises <i>their</i> 3-term quadratic in e^{2x+1} | M1 | FT <i>their</i> 3-term quadratic in e^{2x+1} |
| | $e^{2x+1} = 5$ nfw | A1 | |
| | $x = \frac{-1 + \ln 5}{2}$ oe, isw or 0.305 or 0.3047[18...] isw | A1 | and no other solution |
| | Alternative | | |
| | $e(e^{2x})^2 - 3e^{2x} - 10e^{-1} [= 0]$ oe or $e^2(e^{2x})^2 - 3e(e^{2x}) - 10 [= 0]$ oe or $e^2(e^x)^4 - 3e(e^x)^2 - 10 [= 0]$ oe | (B1) | |
| | Solves or factorises <i>their</i> 3-term quadratic in e^{2x} oe | (M1) | FT <i>their</i> 3-term quadratic in e^{2x} |
| | $e^{2x} = \frac{5}{e}$ or 1.839[...] or $e^x = \sqrt{\frac{5}{e}}$ or 1.356[...] nfw | (A1) | |
| $x = \frac{1}{2} \ln \frac{5}{e}$ oe, isw or 0.305 or 0.3047[18...] isw | (A1) | and no other solution | |
| 6 | $\frac{5 - \sqrt{3}}{(\sqrt{6} + \sqrt{2})^2}$ soi | B1 | |
| | $(\sqrt{6} + \sqrt{2})^2 = 6 + 2 + 2\sqrt{12}$ or $8 + 2\sqrt{12}$ | B1 | |
| | $\frac{5 - \sqrt{3}}{8 + 4\sqrt{3}} \times \frac{8 - 4\sqrt{3}}{8 - 4\sqrt{3}}$ or $\frac{5 - \sqrt{3}}{4(2 + \sqrt{3})} \times \frac{2 - \sqrt{3}}{2 - \sqrt{3}}$ oe | M1 | FT $\frac{a(5 - \sqrt{3})}{b + c\sqrt{d}}$ where a , b , and c are non-zero constants and d is an integer |
| | $\frac{40 - 20\sqrt{3} - 8\sqrt{3} + 12}{8^2 - (4\sqrt{3})^2}$ or $\frac{40 - 20\sqrt{3} - 8\sqrt{3} + 12}{64 - 48}$ or $\frac{10 - 5\sqrt{3} - 2\sqrt{3} + 3}{4(2^2 - (\sqrt{3})^2)}$ or $\frac{10 - 5\sqrt{3} - 2\sqrt{3} + 3}{4}$ | A1 | |
| | $\frac{13}{4} - \frac{7}{4}\sqrt{3}$ oe, nfw | A1 | dep on all previous marks awarded |

| Question | Answer | Marks | Partial marks |
|-----------|--|-----------|---|
| 7(a)(i) | 252 | B1 | |
| 7(a)(ii) | 56 | B2 | B1 for 8C_3 or $\frac{8!}{5! \times 3!}$ oe |
| 7(a)(iii) | 140 | B2 | B1 for ${}^8C_4 \times 2$ oe or ${}^{10}C_5 - {}^8C_5 - {}^8C_3$ oe |
| 7(b) | 483 840 | B3 | B2 for $8! \times 6 [\times 2]$ or 241 920 oe or B1 for $8!$ or 40 320 or $8! \times 2$ or 80 640 |
| 8 | $\operatorname{cosec}^2 2\theta + 3\operatorname{cosec} 2\theta - 10 [= 0]$ or $10\sin^2 2\theta - 3\sin 2\theta - 1 [= 0]$ | B2 | B1 for correctly writing the equation in terms of one trigonometric function e.g. $\operatorname{cosec}^2 2\theta - 1 + 3 \operatorname{cosec} 2\theta = 9$ or $\frac{1 - \sin^2 2\theta}{\sin^2 2\theta} + \frac{3}{\sin 2\theta} = 9$ |
| | Solves or factorises <i>their</i> 3-term quadratic in $\operatorname{cosec} 2\theta$ or $\sin 2\theta$ e.g. $(\operatorname{cosec} 2\theta - 2)(\operatorname{cosec} 2\theta + 5) [= 0]$ or $(2\sin 2\theta - 1)(5\sin 2\theta + 1) [= 0]$ | M1 | FT <i>their</i> 3-term quadratic in $\operatorname{cosec} 2\theta$ or $\sin 2\theta$ |
| | $[\sin 2\theta = \frac{1}{2}$ $\sin 2\theta = -\frac{1}{5}$ $\theta =]$ 15 75 -5.8 or -5.76 to -5.77 -84.2 or -84.23 to -84.232 and no other angles in range; nfw | A3 | A2 for any 2 correct, ignoring extras in range; nfw or A1 for one correct angle or one correct double angle; nfw |

| Question | Answer | Marks | Partial marks |
|----------|--|-----------|--|
| 9(a) | Horizontal line, $v = 6$ for $0 \leq t \leq 5$ | B1 | |
| | Horizontal line, $v = -3$ for $5 \leq t \leq 15$ | B2 | <p>B1 for horizontal line for $5 \leq t \leq 15$ with $v = k$ where $k < 0$ or $v = 3$ for $5 \leq t \leq 15$ or $v = -3$ for $t > 5$ and at least $7 \leq t \leq 13$</p> <p>If 0 scored, award: SC2 for Horizontal line, $v = -6$ for $0 \leq t \leq 5$ and Horizontal line, $v = 3$ for $5 \leq t \leq 15$</p> <p>OR SC1 for Horizontal line, $v = -6$ for $0 \leq t \leq 5$ and Horizontal line for $5 \leq t \leq 15$ with $v = k$ where $k > 0$</p> |
| 9(b) | Single line with positive gradient passing through $(0, -8)$, $(10, 0)$ and $(20, 8)$ | B3 | <p>B2 for a single line with positive gradient</p> <ul style="list-style-type: none"> • passing through a point indicated as $(0, -8)$ or $(20, 8)$ and the point $(10, 0)$ • or passing through points indicated as $(0, -8)$ and $(20, 8)$ which does not pass through $(10, 0)$ <p>or B1 for a single line with positive gradient</p> <ul style="list-style-type: none"> • passing through a point indicated as $(0, -8)$ or $(20, 8)$ to or through any point on the t-axis • or passing through $(10, 0)$ but with both endpoints incorrect or unlabelled <p>If 0 scored, award SC1 for a single line with negative gradient passing through $(0, 8)$, $(10, 0)$ and $(20, -8)$</p> |

| Question | Answer | Marks | Partial marks |
|----------|---|-----------|--|
| 10(a) | Maximum point at (2, 1) oe, nfww and $h = 5$ nfww | B4 | B3 for maximum point at (2, 1) oe or B2 for maximum point when $x = 2$ or B1 for a correct method which could be used to find the maximum point $\left[x\left(1 - \frac{x}{4}\right) = 0 \text{ when } x = 0 \text{ and } \right] x = 4$ or [differentiating and equating to 0:] $1 - \frac{2x}{4} = 0$ or [completing the square to find:] $1 - \frac{1}{4}(x - 2)^2$ If 0 scored, SC1 for $h = 5$ with incorrect or no method shown |
| 10(b) | $x^2 - 4x - 16 [= 0]$ oe or $-\frac{1}{4}(x - 2)^2 = -4 - 1$ oe | B1 | FT <i>their</i> attempt to complete the square, if already seen and of the form $a + b(x + c)^2$ where a , b and c are constants and b is negative |
| | Solves <i>their</i> 3-term quadratic using the formula or completing the square | M1 | FT <i>their</i> rearrangement of $-4 = x - \frac{x^2}{4}$ oe |
| | $x = 2 + 2\sqrt{5}$ oe | A1 | |
| 10(c) | First derivative $1 - \frac{x}{2}$ and substitution of <i>their</i> $2 + 2\sqrt{5}$ or 6.47 | M1 | FT <i>their</i> attempt to differentiate $x - \frac{x^2}{4}$, if already seen, and <i>their</i> $2 + 2\sqrt{5}$ |
| | $1 - 2 \times \frac{2 + 2\sqrt{5}}{4}$ or $-\sqrt{5}$ or awrt -2.24 soi | A1 | dep on correct derivative and correct x -coordinate of A |
| | Correct method to find the angle e.g. $-\tan^{-1}\left(1 - 2 \times \frac{2 + 2\sqrt{5}}{4}\right)$ soi or $\tan^{-1}\sqrt{5}$ or $-\tan^{-1}(-\sqrt{5})$ soi | M1 | dep on previous M1 A1 |
| | Degrees: awrt 65.9 or Radians: awrt 1.15 | A1 | |
| 11(a) | $x = 5$ $y = 5\sqrt{3}$ | B2 | B1 for either component correct |

| Question | Answer | Marks | Partial marks |
|----------|--|-----------|---|
| 11(b) | $\left[\begin{pmatrix} 0 \\ 0 \end{pmatrix} + t \begin{pmatrix} 5 \\ 5\sqrt{3} \end{pmatrix} \right]$ oe, isw | B1 | FT <i>their</i> $\begin{pmatrix} 5 \\ 5\sqrt{3} \end{pmatrix}$, which must be a vector with at least one non-zero component |
| 11(c) | $\begin{pmatrix} 2\sqrt{3} \\ 9 \end{pmatrix} + t \begin{pmatrix} 5 \\ 3 \\ 0 \end{pmatrix}$ oe, isw | B2 | B1 for x component $2\sqrt{3} + \frac{5}{3}t$ seen or y component 9 seen or $\begin{pmatrix} 2\sqrt{3} \\ 9 \end{pmatrix} + t \begin{pmatrix} 5 \\ 3 \\ 0 \end{pmatrix}$ with at most one error but must include t |
| 11(d) | $5t\sqrt{3} = 9$ or $5t = 2\sqrt{3} + \frac{5}{3}t$ | M1 | FT <i>their</i> position vector of A and <i>their</i> position vector of B providing <ul style="list-style-type: none"> both are in terms of t and at least one is of form $\begin{pmatrix} a \\ b \end{pmatrix} + t \begin{pmatrix} c \\ d \end{pmatrix}$ where a, b, c and d are constants |
| | $(5\sqrt{3})t = 9$ and $5t = 2\sqrt{3} + \frac{5}{3}t$ oe | A1 | |
| | Shows the exact times to be the same e.g. $t = \frac{9}{5\sqrt{3}} = \frac{3\sqrt{3}}{5}$ oe and $\frac{10}{3}t = 2\sqrt{3} \rightarrow t = \frac{3\sqrt{3}}{5}$ or $t = \frac{6\sqrt{3}}{10} \rightarrow t = \frac{3\sqrt{3}}{5}$ or $\frac{5}{3}t = \sqrt{3} \rightarrow t = \frac{3\sqrt{3}}{5}$ oe OR Finds a correct value for t , as above, and shows this satisfies the other equation OR Finds a correct value for t , as above, and shows both particles are at $\begin{pmatrix} 9 \\ \sqrt{3} \\ 9 \end{pmatrix}$ oe at this time | A2 | A1 for $t = \frac{9}{5\sqrt{3}}$ and $t = \frac{2\sqrt{3}}{\frac{10}{3}}$ oe |

| Question | Answer | Marks | Partial marks |
|--|---|---|---|
| 12 | Correct use of $x^2h = 5$ to find an expression that can be used to eliminate h | M2 | M1 for $x^2h = 5$ soi |
| | Surface area: $x^2 + 4x\left(\frac{5}{x^2}\right)$ oe | B1 | |
| | Derivative of the surface area: $2x - 20x^{-2}$ oe | B1 | FT <i>their</i> surface area of form $ax^2 + \frac{b}{x}$ |
| | Equates <i>their</i> $2x - 20x^{-2}$ to 0 and solves to find a value of x | M1 | FT <i>their</i> derivative of form $ax + \frac{b}{x^2}$ oe |
| | $x = 2.15$ or $2.154[\dots]$ $h = 1.08$ or $1.077[\dots]$ nfw | A1 | dep on all previous marks awarded |
| | Alternative | | |
| | Correct use of $x^2h = 5$ to find an expression that can be used to eliminate x | (M2) | M1 for $x^2h = 5$ soi |
| Surface area: $\left(\sqrt{\frac{5}{h}}\right)^2 + 4h \times \sqrt{\frac{5}{h}}$ oe | (B1) | | |
| Derivative of the surface area: $4\sqrt{5} \times \frac{1}{2}h^{-\frac{1}{2}} - \frac{5}{h^2}$ oe | (B1) | FT <i>their</i> surface area of form $\frac{a}{h} + b\sqrt{h}$ | |
| Equates <i>their</i> $4\sqrt{5} \times \frac{1}{2}h^{-\frac{1}{2}} - \frac{5}{h^2}$ to 0 and solves to find a value of h | (M1) | FT <i>their</i> derivative of form $\frac{a}{\sqrt{h}} + \frac{b}{h^2}$ oe | |
| $h = 1.08$ or $1.077[\dots]$ $x = 2.15$ or $2.154[\dots]$ nfw | (A1) | dep on all previous marks awarded | |