



Cambridge O Level

CANDIDATE NAME



CENTRE NUMBER

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ADDITIONAL MATHEMATICS

4037/12

Paper 1

October/November 2024

2 hours

You must answer on the question paper.

No additional materials are needed.

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 80.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Any blank pages are indicated.





Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

Arithmetic series $u_n = a + (n-1)d$

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series $u_n = ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

2. TRIGONOMETRY

Identities

$$\begin{aligned} \sin^2 A + \cos^2 A &= 1 \\ \sec^2 A &= 1 + \tan^2 A \\ \operatorname{cosec}^2 A &= 1 + \cot^2 A \end{aligned}$$

Formulae for $\triangle ABC$

$$\begin{aligned} \frac{a}{\sin A} &= \frac{b}{\sin B} = \frac{c}{\sin C} \\ a^2 &= b^2 + c^2 - 2bc \cos A \\ \Delta &= \frac{1}{2}bc \sin A \end{aligned}$$





- 1 The curve $y = a \cos bx + c$, where a , b and c are integers, passes through the points $\left(-\frac{\pi}{6}, -2\right)$ and $\left(\frac{\pi}{9}, \frac{1}{2}\right)$. The curve has a period of $\frac{2\pi}{3}$.
- (a) Find the values of a , b and c . [4]

- (b) Find the least value of y on the curve for $0 \leq x \leq \frac{\pi}{2}$, and state the value of x at which this occurs. [3]





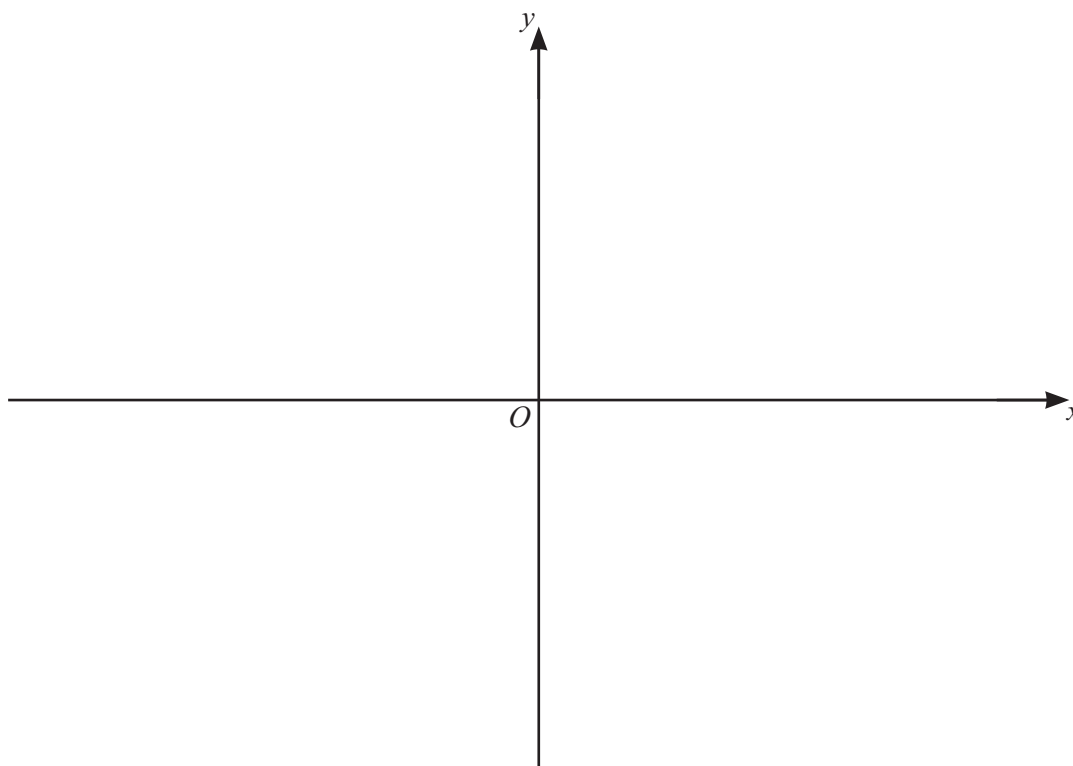
2 It is given that $y = f(x)$, where $f(x) = (2x - 5)(x - 1)^2$.

(a) Find the coordinates of the stationary points on the curve $y = f(x)$.

[4]

(b) On the axes, sketch the graph of $y = f(x)$, stating the intercepts with the axes.

[3]



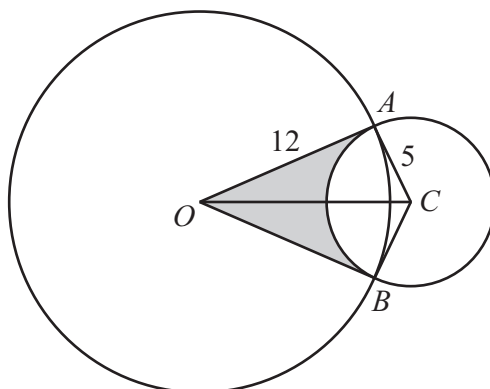
(c) Hence find the values of k for which $f(x) = k$ has exactly one solution.

[2]





- 3 In this question, all lengths are in centimetres and all angles are in radians.



The diagram shows a circle with centre O and radius 12, and a circle with centre C and radius 5. The circles intersect at the points A and B , such that OA and OB are tangents to the circle with centre C .

- (a) Show that the obtuse angle ACB is 2.35 radians, correct to 2 decimal places. [2]
- (b) Find the perimeter of the shaded region. [2]
- (c) Find the area of the shaded region. [3]





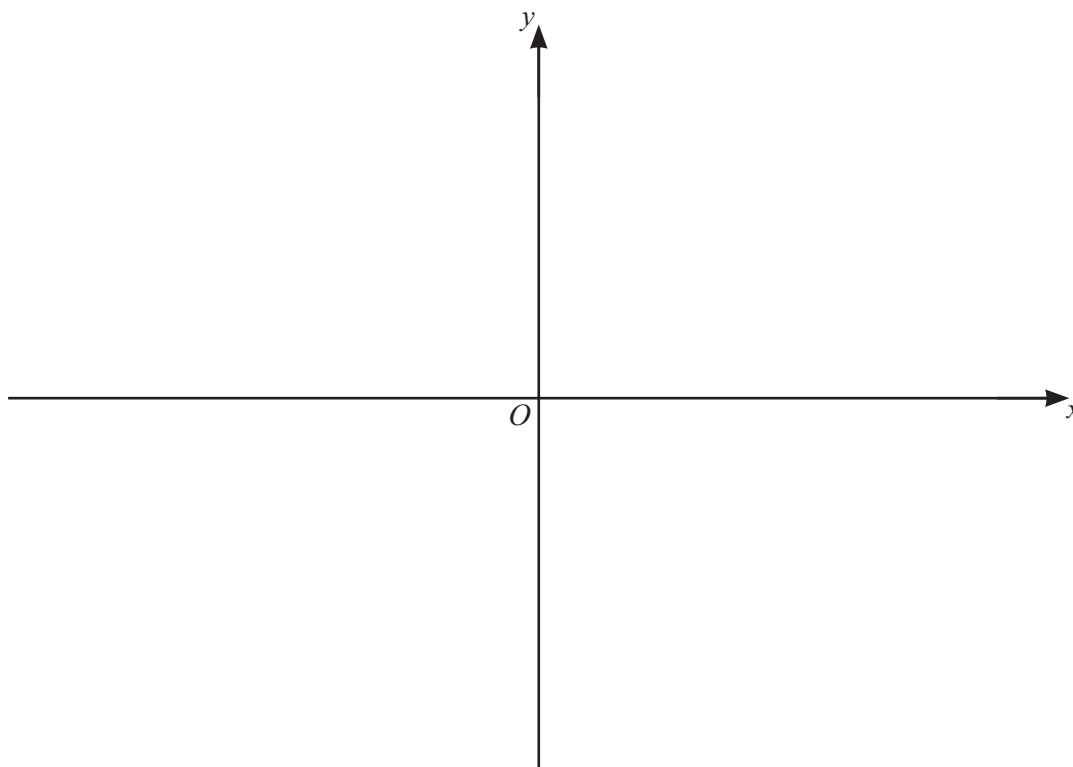
4 The function f is such that $f(x) = 4 \ln(3x - 2)$, for $x > a$, where a is as small as possible.

(a) (i) Write down the value of a . [1]

(ii) Write down the range of f . [1]

(iii) Find $f^{-1}(x)$, stating its domain and range. [4]

(iv) On the axes sketch the graphs of $y = f(x)$ and $y = f^{-1}(x)$, stating the intercepts with the axes. [4]





(b) Given that $g(x) = (2x + 1)^{\frac{1}{2}} + 4$, for $x > 0$, solve the equation $gg(x) = 9$.

[3]

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5 (a) Show that $\frac{1 + \cot^2 \theta}{\cot^2 \theta} = \sec^2 \theta$.

[1]

(b) Write down the derivative of $\tan \theta$ with respect to θ .

[1]

(c) Using **part (a)** and **part (b)**, find the exact value of $\int_0^{\frac{\pi}{3}} \left(\frac{1 + \cot^2 \theta}{\cot^2 \theta} - \sin \theta \right) d\theta$.

[4]





6 (a) Find, in descending powers of x , the first 3 terms in the expansion of $\left(x + \frac{2}{x^2}\right)^{10}$. Simplify each term as far as possible. [3]

(b) Find the term independent of x in the expansion of $\left(4x^2 + \frac{1}{2x^2}\right)^8$. [2]



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7 It is given that $y = \frac{\ln(3x^2 - 1)}{x + 2}$, for $x > \frac{1}{\sqrt{3}}$. When $x = 1$, y is increasing at the rate of h units per second. Find, in terms of h , the corresponding rate of change in x , giving your answer in exact form.

[6]

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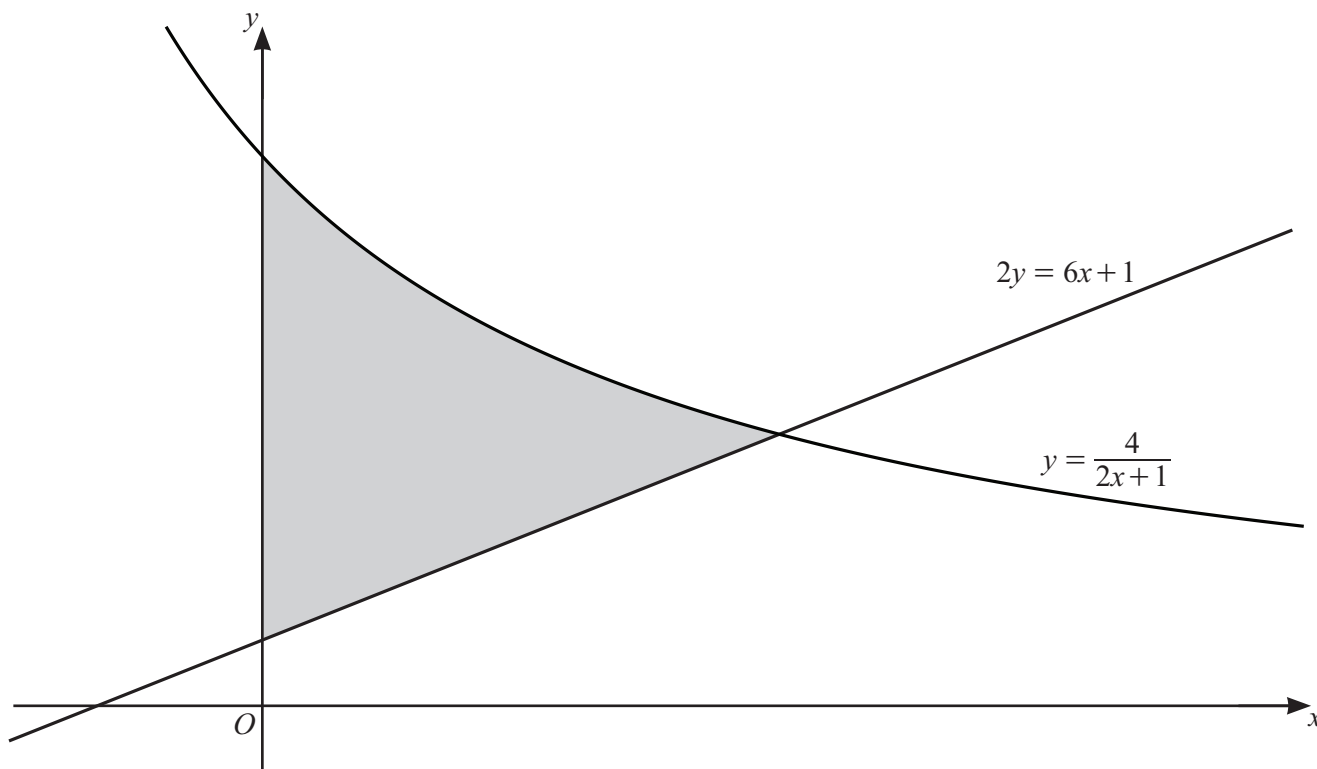
- 8 The tangent to the curve $y = e^x(2x + 5)^{\frac{1}{2}}$ at the point where $x = 2$ meets the x -axis at the point X and the y -axis at the point Y . Find the coordinates of the mid-point of XY , giving your answer in exact form. [8]

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9



The diagram shows part of the curve $y = \frac{4}{2x+1}$ and the straight line $2y = 6x + 1$. Find the area of the shaded region, giving your answer in the form $\ln a + b$, where a is an integer and b is a rational number. [8]





Continuation of working space for question 9.

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- 10 (a) The first 3 terms of an arithmetic progression are $2 \tan 2x$, $5 \tan 2x$, $8 \tan 2x$. Find the values of x , where $-180^\circ \leq x \leq 180^\circ$, for which the sum to 30 terms is $455\sqrt{3}$. [5]

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(b) The first 3 terms of a geometric progression are

$$5 \cos^2\left(\theta - \frac{\pi}{2}\right), \quad 20 \cos^4\left(\theta - \frac{\pi}{2}\right), \quad 80 \cos^6\left(\theta - \frac{\pi}{2}\right), \quad \text{where } -\frac{\pi}{6} \leq \theta \leq \frac{7\pi}{6}.$$

Find the values of θ for which this geometric progression has a sum to infinity.

[6]

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