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Cambridge Assessment International Education

Cambridge O Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

* 0 4 4 6 7 1 3 0 2 3 *

ADDITIONAL MATHEMATICS **4037/12**

Paper 1 **October/November 2024**

2 hours

You must answer on the question paper.

No additional materials are needed.

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 80.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

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Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!\,r!}$

Arithmetic series $\quad u_n = a + (n-1)d$

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series $\quad u_n = ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}bc\sin A$$

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1 The curve $y = a\cos bx + c$, where a, b and c are integers, passes through the points $\left(-\frac{\pi}{6}, -2\right)$ and $\left(\frac{\pi}{9}, \frac{1}{2}\right)$. The curve has a period of $\frac{2\pi}{3}$.

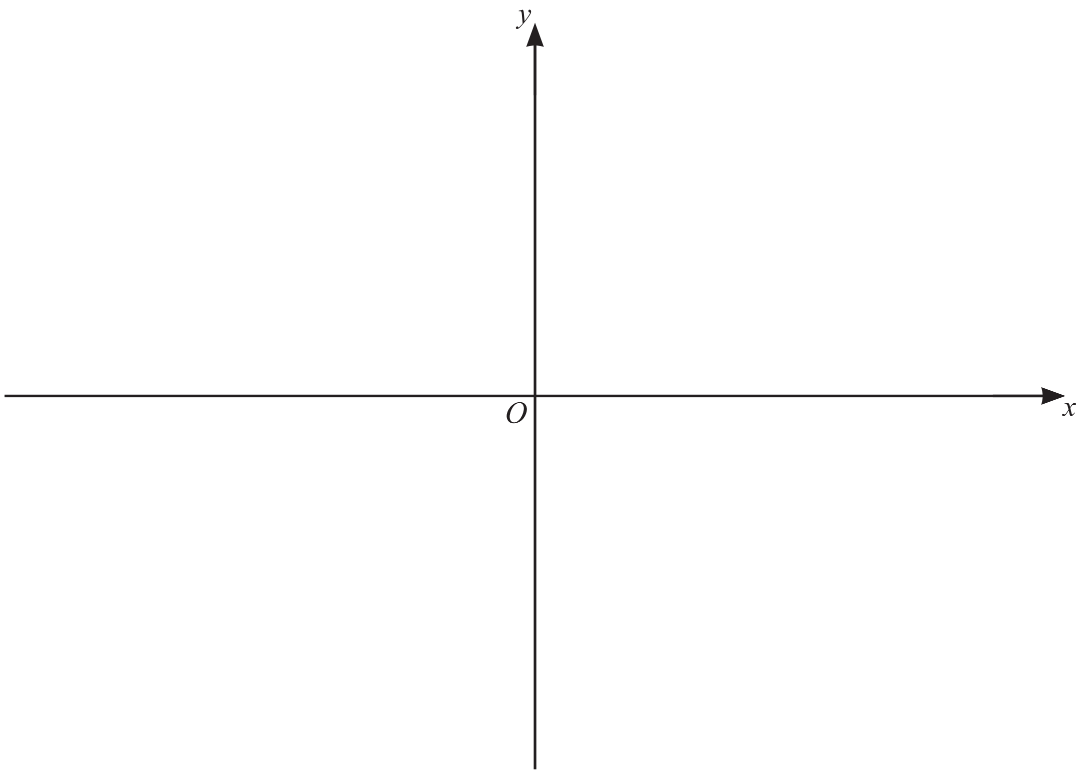
(a) Find the values of a, b and c. [4]

(b) Find the least value of y on the curve for $0 \leqslant x \leqslant \frac{\pi}{2}$, and state the value of x at which this occurs. [3]

2 It is given that $y = f(x)$, where $f(x) = (2x-5)(x-1)^2$.

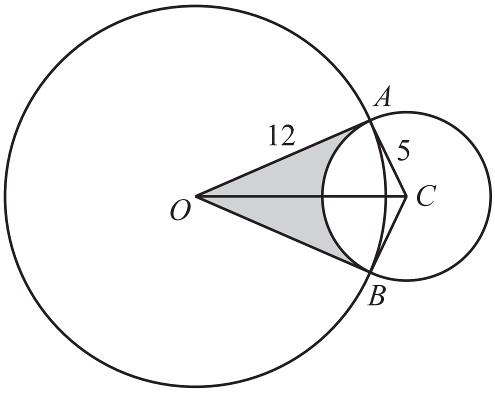
(a) Find the coordinates of the stationary points on the curve $y = f(x)$. [4]

(b) On the axes, sketch the graph of $y = f(x)$, stating the intercepts with the axes. [3]



(c) Hence find the values of k for which $f(x) = k$ has exactly one solution. [2]

3 In this question, all lengths are in centimetres and all angles are in radians.



The diagram shows a circle with centre O and radius 12, and a circle with centre C and radius 5. The circles intersect at the points A and B, such that OA and OB are tangents to the circle with centre C.

(a) Show that the obtuse angle ACB is 2.35 radians, correct to 2 decimal places. [2]

(b) Find the perimeter of the shaded region. [2]

(c) Find the area of the shaded region. [3]

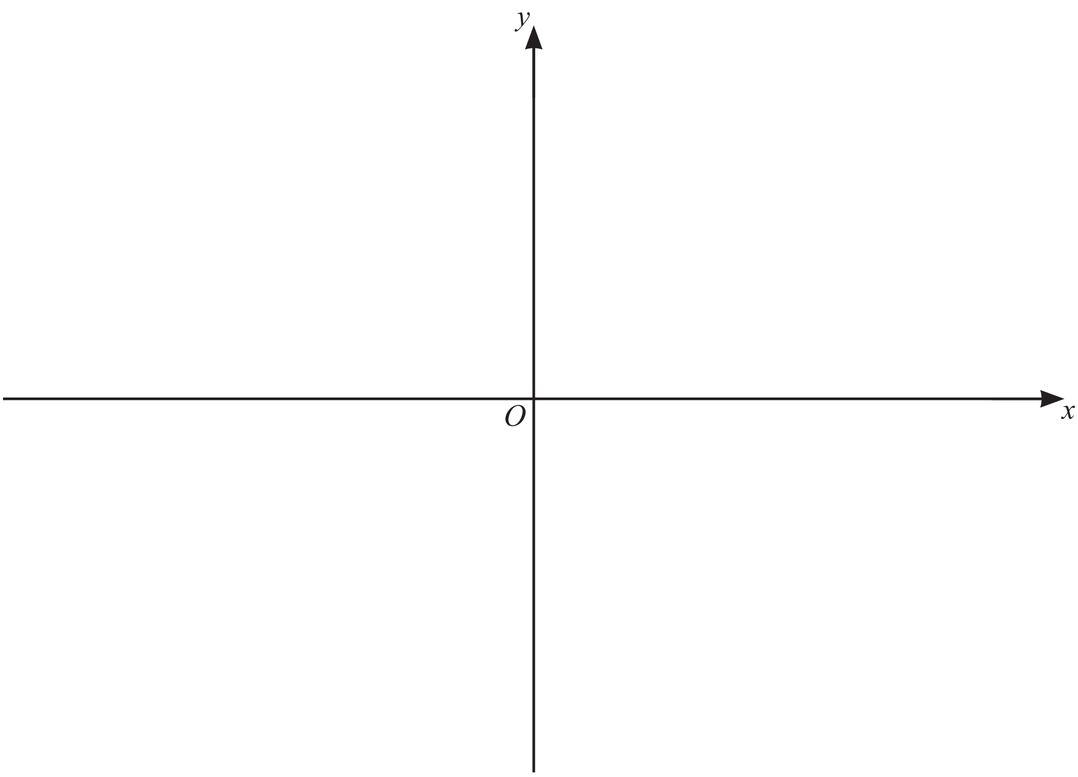
4 The function f is such that $f(x) = 4\ln(3x-2)$, for $x > a$, where a is as small as possible.

(a) **(i)** Write down the value of a. [1]

(ii) Write down the range of f. [1]

(iii) Find $f^{-1}(x)$, stating its domain and range. [4]

(iv) On the axes sketch the graphs of $y = f(x)$ and $y = f^{-1}(x)$, stating the intercepts with the axes. [4]



(b) Given that $g(x) = (2x+1)^{\frac{1}{2}} + 4$, for $x > 0$, solve the equation $gg(x) = 9$. [3]

5 **(a)** Show that $\dfrac{1+\cot^2\theta}{\cot^2\theta} = \sec^2\theta$. [1]

(b) Write down the derivative of $\tan\theta$ with respect to θ. [1]

(c) Using **part (a)** and **part (b)**, find the exact value of $\displaystyle\int_0^{\frac{\pi}{3}} \left(\frac{1+\cot^2\theta}{\cot^2\theta} - \sin\theta\right) d\theta$. [4]

6 (a) Find, in descending powers of x, the first 3 terms in the expansion of $\left(x+\frac{2}{x^2}\right)^{10}$. Simplify each term as far as possible. [3]

(b) Find the term independent of x in the expansion of $\left(4x^2+\frac{1}{2x^2}\right)^8$. [2]

7 It is given that $y = \dfrac{\ln(3x^2 - 1)}{x+2}$, for $x > \dfrac{1}{\sqrt{3}}$. When $x = 1$, y is increasing at the rate of h units per second. Find, in terms of h, the corresponding rate of change in x, giving your answer in exact form. [6]

8 The tangent to the curve $y = e^x(2x+5)^{\frac{1}{2}}$ at the point where $x = 2$ meets the x-axis at the point X and the y-axis at the point Y. Find the coordinates of the mid-point of XY, giving your answer in exact form. [8]

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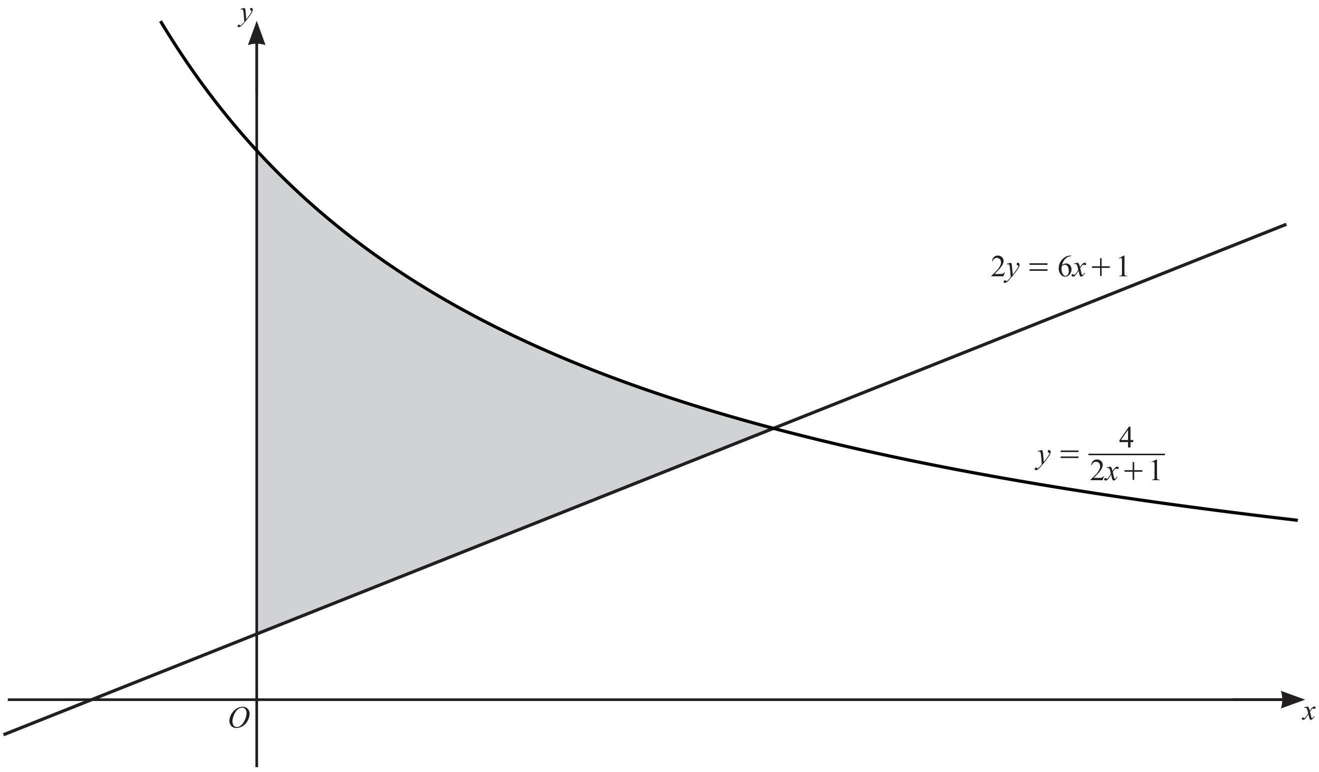

The diagram shows part of the curve $y = \dfrac{4}{2x+1}$ and the straight line $2y = 6x + 1$. Find the area of the shaded region, giving your answer in the form $\ln a + b$, where a is an integer and b is a rational number. [8]

Continuation of working space for question 9.

10 **(a)** The first 3 terms of an arithmetic progression are $2\tan 2x$, $5\tan 2x$, $8\tan 2x$. Find the values of x, where $-180° \leqslant x \leqslant 180°$, for which the sum to 30 terms is $455\sqrt{3}$. [5]

(b) The first 3 terms of a geometric progression are

$$5\cos^2\left(\theta-\frac{\pi}{2}\right),\quad 20\cos^4\left(\theta-\frac{\pi}{2}\right),\quad 80\cos^6\left(\theta-\frac{\pi}{2}\right),\quad \text{where } -\frac{\pi}{6}\leqslant\theta\leqslant\frac{7\pi}{6}.$$

Find the values of θ for which this geometric progression has a sum to infinity. [6]

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