Cambridge Assessment International Education

Cambridge O Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

8611364537

ADDITIONAL MATHEMATICS **4037/12**

Paper 1 **May/June 2024**

2 hours

You must answer on the question paper.

No additional materials are needed.

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 80.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

Arithmetic series $u_n = a + (n-1)d$

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series $u_n = ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

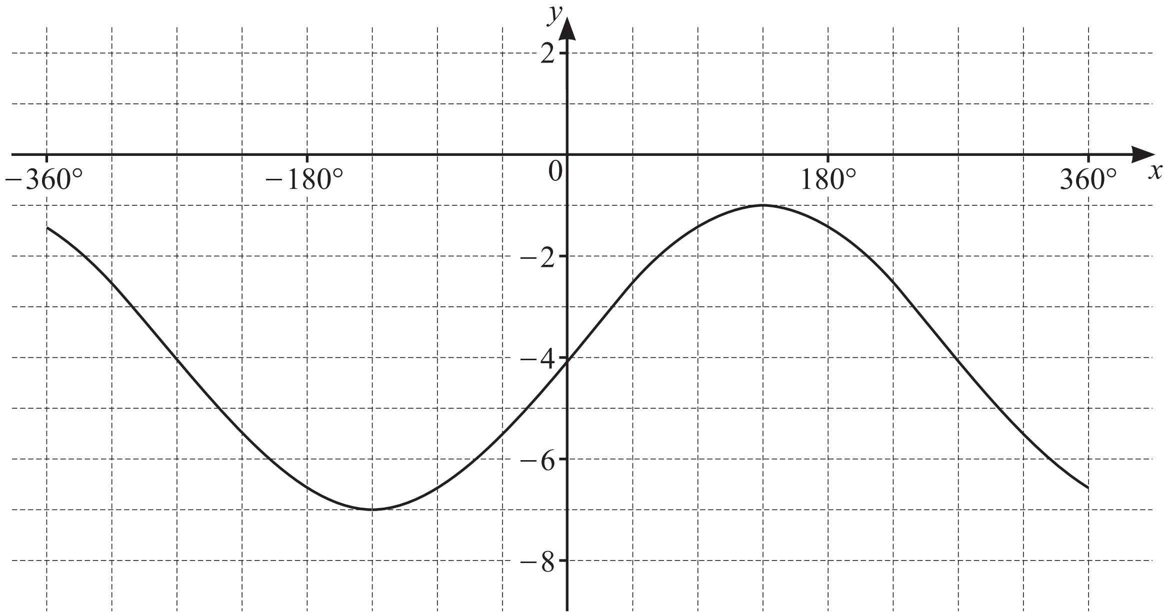
Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}bc\sin A$$

1



The diagram shows the graph of $y = a\sin bx + c$ for $-360^\circ \leqslant x \leqslant 360^\circ$, where a, b and c are constants. Find the values of a, b and c. [3]

2 Given that $\log_3 r + 2\log_9 s = 8$, find the value of rs. [3]

3 Given that $y = \tan\dfrac{x}{2}$, find the exact value of $\dfrac{dy}{dx}$ when $x = \dfrac{\pi}{3}$. [4]

4 A team of 8 people is to be formed from 6 teachers, 5 doctors and 4 police officers.

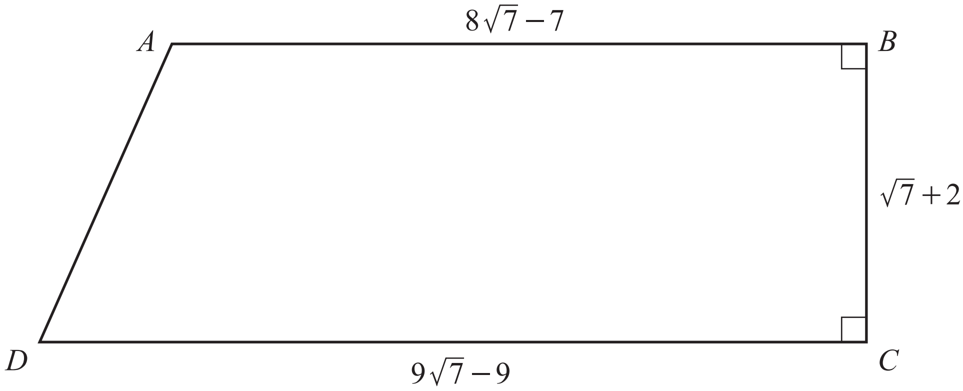
(a) Find the number of teams that can be formed. [1]

(b) Find the number of teams that can be formed without any teachers. [1]

(c) Find the number of teams that can be formed with the same number of doctors as teachers. [4]

5 DO NOT USE A CALCULATOR IN THIS QUESTION.

In this question, all lengths are in centimetres.



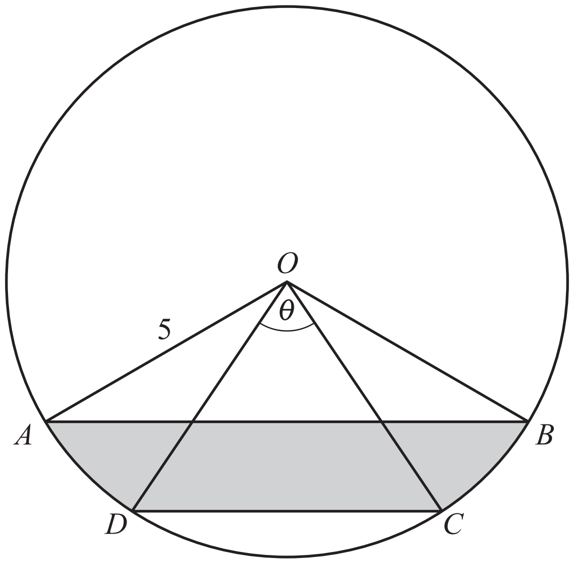
The diagram shows the trapezium *ABCD*. The lengths of *AB*, *BC* and *CD* are $8\sqrt{7}-7$, $\sqrt{7}+2$ and $9\sqrt{7}-9$ respectively. The line *BC* is perpendicular to the lines *AB* and *CD*.

(a) Find the perimeter of the trapezium, giving your answer in its simplest form. [3]

(b) Find the area of the trapezium, giving your answer in the form $p\sqrt{7}+q$, where p and q are rational numbers. [3]

(c) Find $\cot DBC$, giving your answer in the form $r\sqrt{7}+s$, where r and s are simplified rational numbers. [3]

6 In this question, all lengths are in metres and all angles are in radians.



The diagram shows a circle with centre O and radius 5. The points A, B, C and D lie on the circumference of the circle. Angle $DOC = \theta$. Angle AOD = angle $COB = 0.5$. The length of the minor arc DC is 3.75.

(a) Show that $\theta = 0.75$. [1]

(b) Find the perimeter of the shaded region. [5]

(c) Find the area of the shaded region. [3]

7 **(a)** The line $y = 3x - 2$ intersects the curve $2x^2 - xy + y^2 = 2$ at the points A and B. The point C with coordinates $\left(k, \frac{7}{8}\right)$ lies on the perpendicular bisector of the line AB. Find the exact value of k. [9]

(b) The point D lies on the perpendicular bisector of AB such that D is a reflection of C in the line AB. Find the coordinates of D. [2]

8 A curve has equation $y = \dfrac{(3x^2-5)^{\frac{1}{3}}}{x+4}$.

(a) Show that $\dfrac{dy}{dx}$ can be written in the form $\dfrac{Ax^2+Bx+C}{(3x^2-5)^{\frac{2}{3}}(x+4)^2}$, where A, B and C are integers. [5]

(b) Hence find the x-coordinates of the stationary points on the curve. Give your answers in their simplest exact form. [3]

9 In this question, all distances are in metres and time, t, is in seconds.

A particle P moves with a speed of 14.5 parallel to the vector $\begin{pmatrix} -20 \\ 21 \end{pmatrix}$.

(a) Find the velocity vector of P. [2]

Initially, P has position vector $\begin{pmatrix} 3 \\ 5 \end{pmatrix}$.

(b) Write down the position vector of P at time t. [2]

A second particle Q has position vector $\begin{pmatrix} -1 \\ 3 \end{pmatrix} + \begin{pmatrix} -5 \\ 7.5 \end{pmatrix} t$ at time t.

(c) Find, in terms of t, the distance between P and Q at time t. Simplify your answer. [4]

(d) Hence show that P and Q never collide. [2]

10 **(a)** The first 3 terms of an arithmetic progression are $3\sin 2x$, $5\sin 2x$, $7\sin 2x$.

(i) Show that the sum to n terms of this arithmetic progression can be written in the form $n(n+a)\sin 2x$, where a is a constant. [3]

(ii) Given that $x = \frac{2\pi}{3}$, find the exact sum of the first 20 terms. [2]

(b) The first 3 terms of a geometric progression are $\ln 2y$, $\ln 4y^2$, $\ln 16y^4$.

(i) Find the nth term of this geometric progression. [2]

(ii) Find the sum to n terms of this geometric progression, giving your answer in its simplest form. [2]

(c) The first 3 terms of a different geometric progression are $\left(2w-\frac{1}{4}\right)$, $\left(2w-\frac{1}{4}\right)^2$, $\left(2w-\frac{1}{4}\right)^3$.

Find the values of w for which this geometric progression has a sum to infinity. [3]

Question 11 is printed on the next page.

11 **(a)** Given that $y = x^2 \ln x$, find $\frac{dy}{dx}$. [2]

(b) Hence find $\int x \ln x \, dx$. [3]