Cambridge O Level

Cambridge Assessment International Education
Cambridge Ordinary Level

ADDITIONAL MATHEMATICS **4037/21**

Paper 2 **May/June 2019**

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2019 series for most Cambridge IGCSE™, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

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Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method marks, awarded for a valid method applied to the problem.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.

B Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation '**dep**' is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

Abbreviations

awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
nfww	not from wrong working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied

Question	Answer	Marks	Partial Marks
1	$6x^2 + 7x - 20$ [*0]	**M1**	where * may be any inequality sign or =
	Critical values $\frac{4}{3}$, $-\frac{5}{2}$	**A1**	
	$x \leqslant -\frac{5}{2}$ or $x \geqslant \frac{4}{3}$ final answer	**A1**	**FT** *their* critical values using outside regions
2(i)	$\frac{\mathrm{d}}{\mathrm{d}x}(\ln x) = \frac{1}{x}$ soi	**B1**	
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{x^3\left(\frac{1}{x}\right) - 3x^2 \ln x}{\left(x^3\right)^2}$ or $x^{-3}\left(\frac{1}{x}\right) + \left(-3x^{-4}\right)\ln x$	**M1**	
	Completion to given answer: $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1 - 3\ln x}{x^4}$	**A1**	
2(ii)	$\left(\frac{1 - 3\ln \mathrm{e}}{\mathrm{e}^4}\right)h$	**M1**	
	$-\frac{2h}{\mathrm{e}^4}$ oe or $-0.0366h$ awrt	**A1**	
3(i)	Correct shape 0.6 oe indicated on x-axis 3 indicated on y-axis	**3**	**B1** correct shape must have cusp on x-axis **B1** for each correct point There must be a sketch to award the marks for the intercepts and sketch should be continuous with one intersection only on each axis
3(ii)	Solves $5x - 3 = x - 2$ oe or $(5x-3)^2 = (2-x)^2$	**M1**	
	$[x =] \frac{1}{4}$ oe	**A1**	
	$[x =] \frac{5}{6}$ oe	**B1**	

Question	Answer	Marks	Partial Marks
4	$(\sqrt{5}-3)^2 = 5+9-2(3)\sqrt{5}$	**M1**	
	$\frac{their(14-6\sqrt{5})}{\sqrt{5}+1} \times \frac{\sqrt{5}-1}{\sqrt{5}-1}$	**M1**	Attempts to rationalise or forms a pair of simultaneous equations e.g. $5p+q=14, \quad p+q=-6$
	$\frac{their(14\sqrt{5}-30-14+6\sqrt{5})}{5-1}$	**M1**	multiplies out; numerator must have at least 3 terms; condone one sign error in numerator; denominator may be 4 or $5-\sqrt{5}+\sqrt{5}-1$ or solves *their* simultaneous equations to find one unknown
	$5\sqrt{5}-11$	**A1**	or $p=5, q=-11$
5(i)	$-\frac{10}{6}$ oe	**B1**	
5(ii)	27	**B1**	
5(iii)	Attempts to find total area	**M1**	
	$\frac{1}{2}(23+their\ k+6)\times 10$ or $\frac{1}{2}\times 4\times 10+23\times 10+\frac{1}{2}\times 6\times 10$	**M1**	
	280	**A1**	
6(a)	$(x+3)(x-3)-2x(-x)$	**B1**	
	their $\det \mathbf{A} = 0$	**M1**	Can be implied by later work
	$[x=] \quad \pm\sqrt{3}$ isw	**A1**	
6(b)(i)	3 × 2 or 3 by 2	**B1**	
6(b)(ii)	**BC** is a 3 by 3 matrix and **CB** is a 2 by 2 matrix [so they cannot be the same] oe or [**CB** =] $\begin{pmatrix} 6 & 5 \\ 41 & 15 \end{pmatrix}$ [so not equal] or finding one correct element of **CB** as being different from **BC and** commenting that the elements are different, [the matrices cannot be the same] oe	**B2**	**B1** for a partially correct statement e.g. The orders are not the same **or BC** is a 3 by 3 matrix **or CB** is a 2 by 2 matrix or **B1** for 3 correct elements or **B1** for finding one correct element of **CB** as being different from **BC**, without further comment

Question	Answer	Marks	Partial Marks
7(i)	$\sec^2 u$	**B1**	
7(ii)	Attempts $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$ or $\frac{dy}{dx} = \frac{dy}{du} \div \frac{dx}{du}$	**M1**	
	$\frac{dy}{dx} = \frac{their \sec^2 u}{3u^2}$	**A1**	**FT** *their* **(i)**
	$u = \sqrt[3]{x-1}$ soi	**B1**	
	$\frac{\sec^2(\sqrt[3]{x-1})}{3(\sqrt[3]{x-1})^2}$ cao	**A1**	final answer If **B1** only then **SC1** for $k(x-1)^{-\frac{2}{3}} \sec^2(x-1)^{\frac{1}{3}}$
8(i)	[angle ECD =] $\frac{5\pi}{18}$ oe or 0.873 soi	**B1**	
	Attempts to find AC and subtract 8	**M1**	e.g. $AC = \frac{8}{\cos\frac{2\pi}{9}}$
	[DC =] 2.44	**A1**	
	$\frac{1}{2} \times 8 \times their AC \times \sin\frac{2\pi}{9}$ OR $\frac{1}{2} \times 8 \times 8\tan\left(\frac{2\pi}{9}\right) - \frac{1}{2} \times 8^2 \times \frac{2\pi}{9}$ $- \frac{1}{2} \times their\, 2.44^2 \times their \frac{5\pi}{18}$	**M2**	**M1** for $\frac{1}{2} \times 8^2 \times \frac{2\pi}{9}$ or for $\frac{1}{2} \times their\, 2.44^2 \times their \frac{5\pi}{18}$ seen
	awrt 1.91	**A1**	
8(ii)	$their(6.712 - 2.443)$ $+ their\, 2.443\left(\frac{5\pi}{18}\right) + 8\left(\frac{2\pi}{9}\right)$	**M2**	**M1** for either arc seen
	awrt 12.0	**A1**	
9(a)(i)	39 916 800	**B1**	
9(a)(ii)	$5! \times 6!$ oe	**M1**	
	86 400	**A1**	

Question	Answer	Marks	Partial Marks
9(b)(i)	${}^5C_3 \times {}^3C_1$ oe	**M1**	
	30	**A1**	
9(b)(ii)	${}^5C_2 \times {}^3C_2 + {}^5C_1 \times {}^3C_1$ oe	**M1**	
	45	**A1**	
10(i)	$\frac{4-3}{1-p} = \frac{1}{3}$ oe	**M1**	**ALT** uses $y = mx + c$ with A and B as far as an equation in p only
	–2	**A1**	
10(ii)	**Either:** Finds midpoint AB $\left(\frac{their\ p+1}{2}, \frac{3+4}{2}\right)$	**B1**	**FT** *their p*
	Verifies $(-0.5, 3.5)$ is on L	**B1**	
	$y = -3x + 2$ therefore $m = -3$ oe **and** $\frac{1}{3} \times -3 = -1$ oe	**B1**	
	Or: finds midpoint AB $\left(\frac{their\ p+1}{2}, \frac{3+4}{2}\right)$	**B1**	**FT** *their p*
	$\frac{1}{3} \times -3 = -1$ oe	**B1**	
	$y - 3.5 = -3(x + 0.5)$ **and** completion to $y = -3x + 2$	**B1**	
10(iii)	$q = 4$	**B1**	
10(iv)	22.5 nfww	**B2**	**B1** for correct method to find area using correct values e.g. $\frac{1}{2} \times AB \times MC$ where M is the midpoint of AB

Question	Answer	Marks	Partial Marks
11(a)(i)	$\frac{1}{\sin\theta}\left(\frac{1}{\sin\theta}-\frac{\cos\theta}{\sin\theta}\right)$	**M2**	**M1** for either $\frac{\operatorname{cosec}\theta-\cot\theta}{\sin\theta}=\frac{1}{\sin\theta}\left(\operatorname{cosec}\theta-\frac{\cos\theta}{\sin\theta}\right)$ or $\frac{\operatorname{cosec}\theta-\cot\theta}{\sin\theta}=\frac{1}{\sin\theta}\left(\frac{1}{\sin\theta}-\cot\theta\right)$
	$\frac{1-\cos\theta}{1-\cos^2\theta}$	**M1**	
	$\frac{1-\cos\theta}{(1-\cos\theta)(1+\cos\theta)}=\frac{1}{1+\cos\theta}$	**A1**	
11(a)(ii)	awrt 233.1	**B2**	with no extras in range **B1** for $\cos\theta=-\frac{3}{5}$ soi
11(b)	$3\phi-4=\tan^{-1}\left(-\frac{1}{2}\right)$ soi	**M1**	
	awrt 0.132, 1.18	**A2**	with no extras in range **A1** for one correct
12(a)	$\frac{e^{2x}}{2}$ seen	**B1**	
	$\frac{e^{2a}}{2}-\frac{1}{2}=50$	**M1**	Uses limits correctly for their integral and sets = 50
	Rearranges and takes logs to base e: $2a=\ln 101$ oe	**M1**	Using *their* integral
	$a=\frac{1}{2}\ln 101$ or $\ln\sqrt{101}$ final answer	**A1**	Allow any exact equivalent
12(b)(i)	$[y=]\,3x-\frac{2}{5}\sin 5x\ [+c]$	**B2**	**B1** for $-k\sin 5x$ where $k>0$
	$\frac{8\pi}{5}=\frac{3\pi}{5}-\frac{2}{5}\sin\left(5\times\frac{\pi}{5}\right)+c$	**M1**	
	$y=3x-\frac{2}{5}\sin 5x+\pi$	**A1**	

Question	Answer	Marks	Partial Marks
12(b)(ii)	$\left[\int y\,\mathrm{d}x = \int\left(3x - \frac{2}{5}\sin 5x + \pi\right)\mathrm{d}x\right]$ $= \frac{3x^2}{2} + \frac{2}{25}\cos 5x + \pi x\,[+c]$	**B3**	**B2** for $\frac{2}{25}\cos 5x$ oe nfww and **B1FT** for $\frac{3x^2}{2} + \ldots + \pi x\,[+c]$
	their $\mathrm{F}(\pi)$ – *their* $\mathrm{F}\left(\frac{\pi}{2}\right)$	**M1**	
	16[.0] or 15.95 to 15.96 or $\frac{13\pi^2}{8} - \frac{2}{25}$	**A1**	